Scaling Up DPLL(T) String Solvers Using Context-Dependent Simplification

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Importance of String Solvers

- Automated string solvers are essential for formal methods applications
- Security applications (e.g. finding XSS attacks) require *string solvers* that:
  - Are *highly efficient*
  - Reason about strings with *unbounded length* (not just bounded ones)
  - Accept a *rich language* of string constraints

In This Paper:

- DPLL(T) string solvers for *extended string constraints*
- New technique, *context-dependent simplification*, improves *scalability* of current string solvers
- Implemented in SMT solver *CVC4*
- *Experiments* show advantages using CVC4 as backend to *symbolic execution* engine PyEx

Basic String Constraints

- Equalities and disequalities between:
  - *Basic string* terms
    - String constants: $\in$, “abc”
    - Concatenation: $x \cdot$“abc”
    - Length: $|\ x\ |$
  - *Linear arithmetic* terms: $x+4$, $y>2$

Example: $x \cdot \text{“a”} = y \wedge |y| > |x| + 2$

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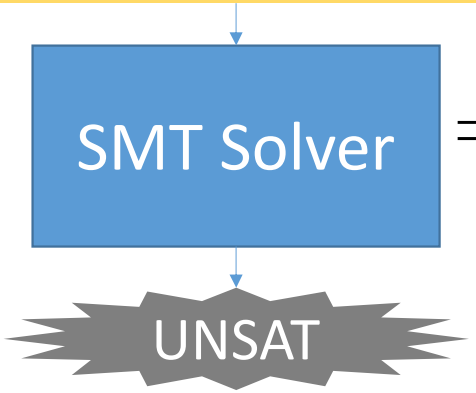


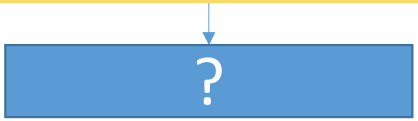
$\Rightarrow$ *Procedure for basic constraints in CVC4*

[Liang et al CAV2014]

Extended String Constraints

- Equalities and disequalities between:
  - *Basic string* terms
    - String constants: $\in$, “abc”
    - Concatenation: x·“abc”
    - Length: | x |
  - *Linear arithmetic* terms: x+4, y>2
  - *Extended* string terms:
    - Substring: substr(“abcde”,1,3)
    - String contains: contains(“abcde”,”cd”)
    - Find “index of”: indexof(“abcde”,”d”,0)
    - String replace: replace(x,“a”,“b”)

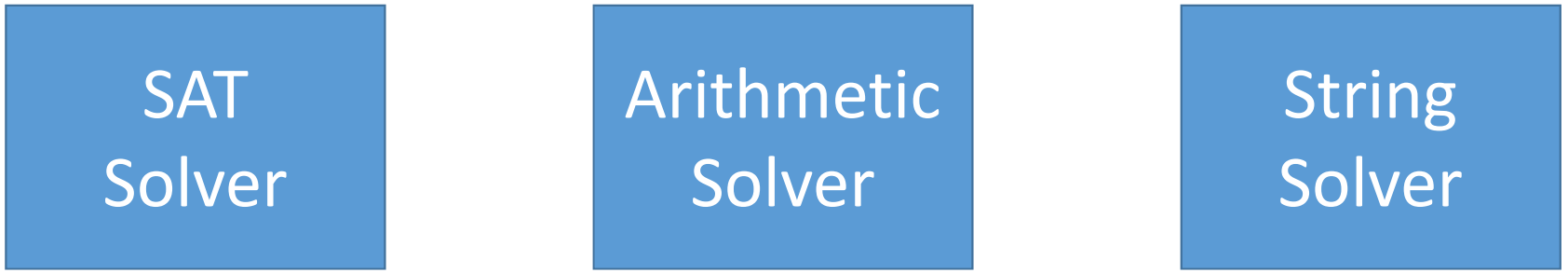
Example: $\dot{E}$contains(substr(x,0,3),“a”) $\wedge$ 0$\leq$indexof(x,“ab”,0)<4



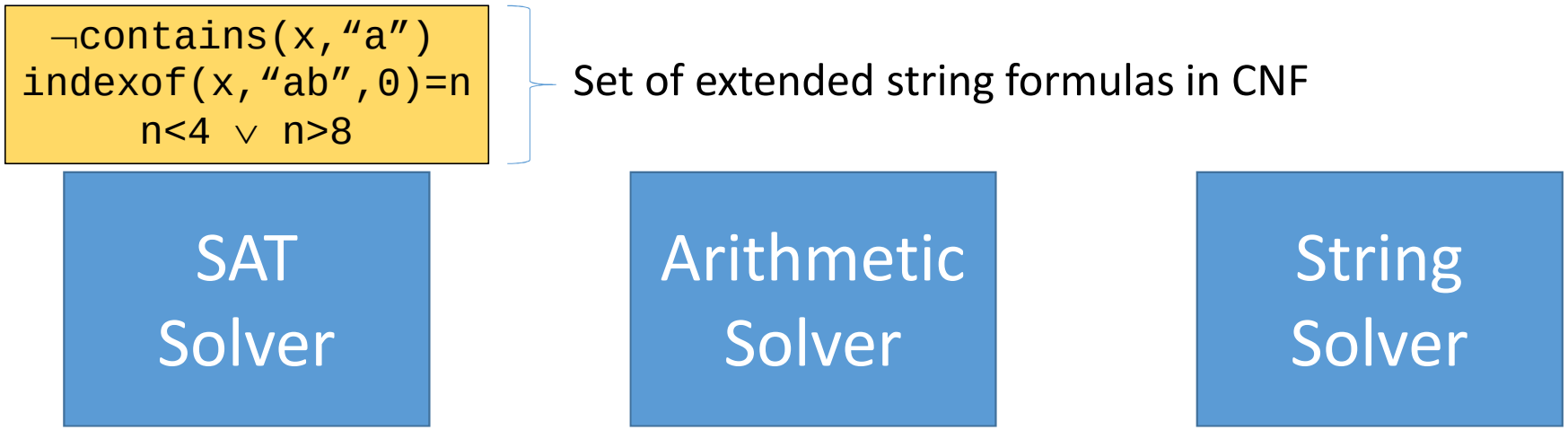
$\Rightarrow$ *Focus of this work*

DPLL(T) String Solvers

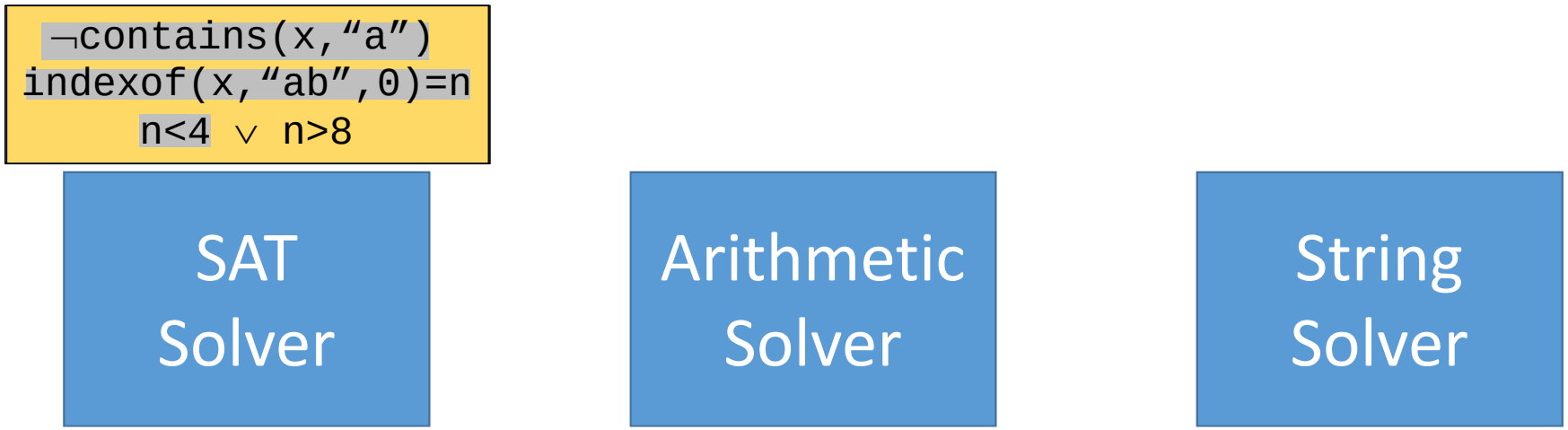
- Cooperation between:



DPLL(T) String Solvers

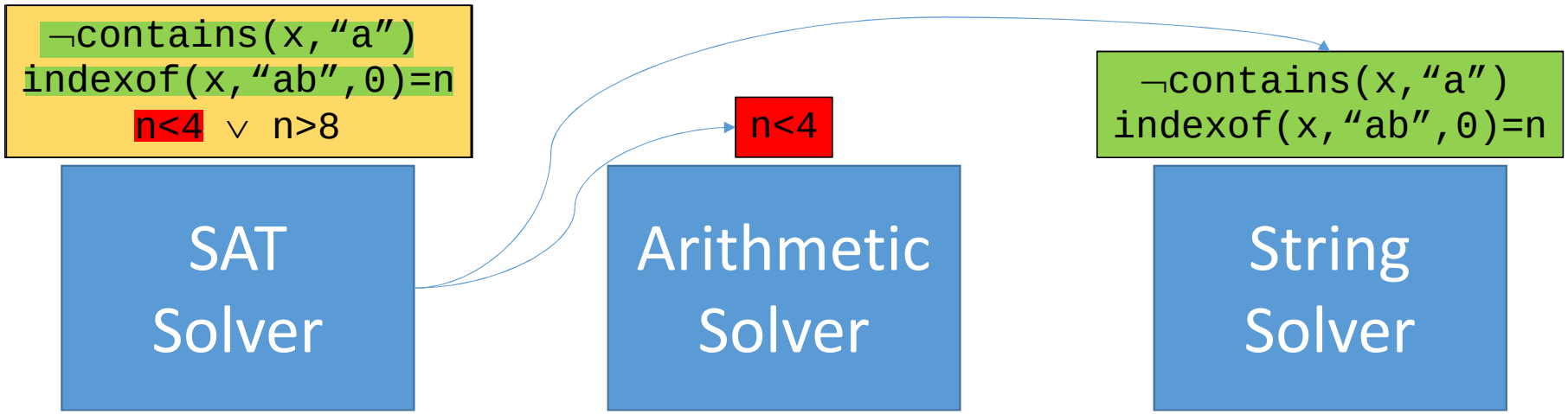


DPLL(T) String Solvers



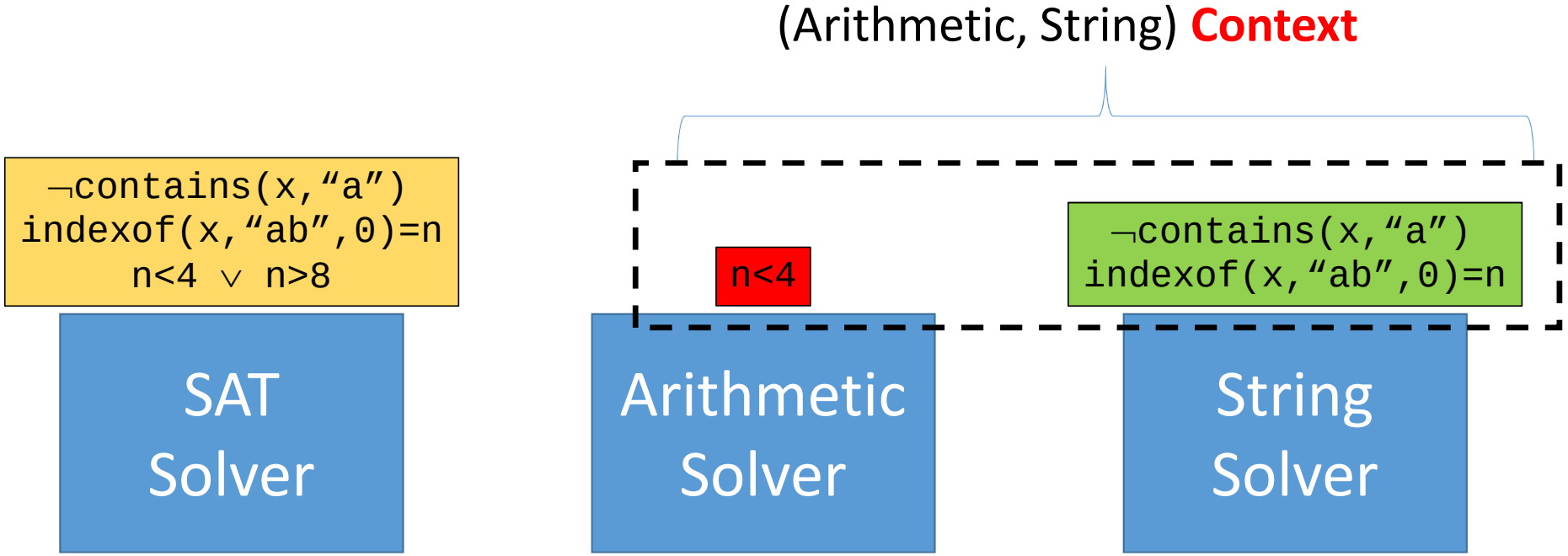
⇒ Find propositionally satisfying assignment

DPLL(T) String Solvers



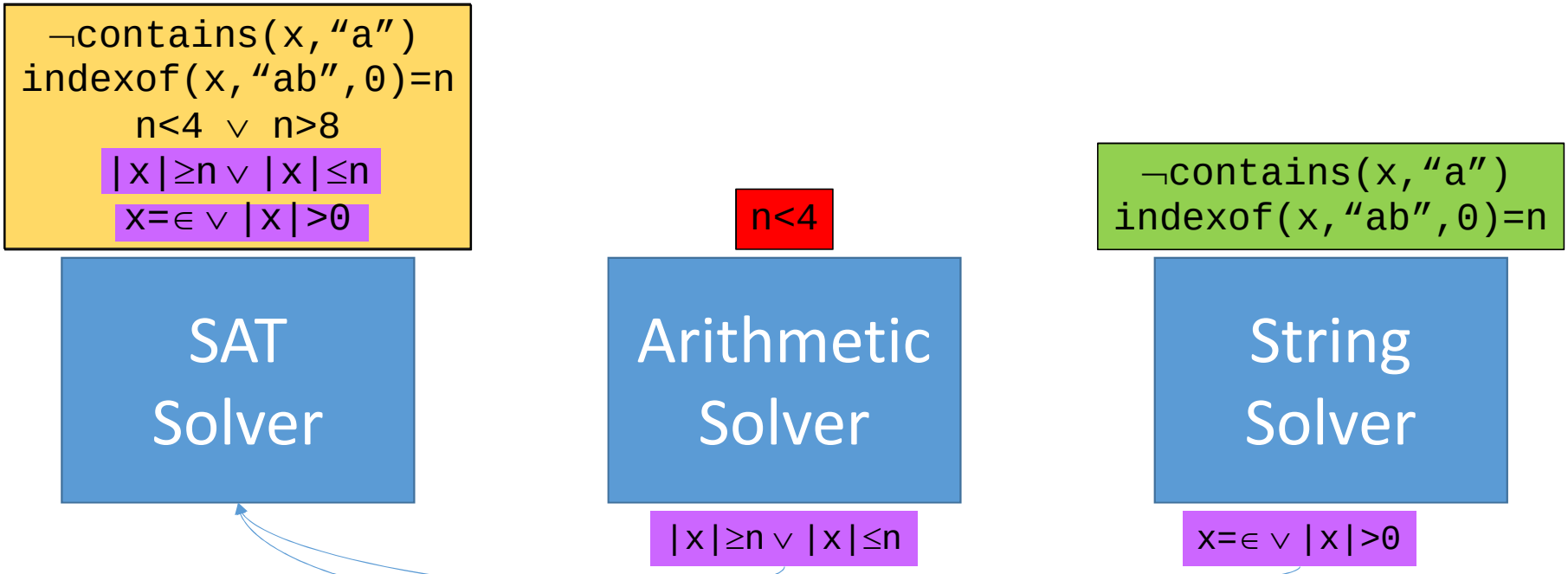
⇒ Distribute to arithmetic and string solvers

DPLL(T) String Solvers



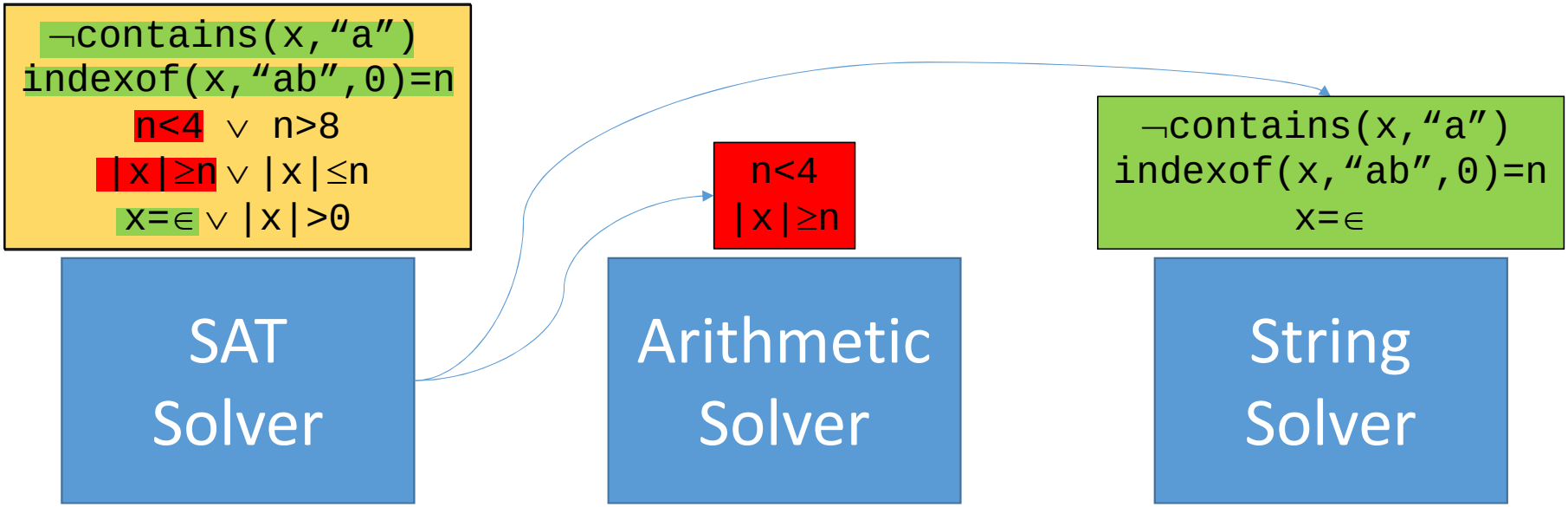
⇒ Solvers maintain a **context** (conjunction of theory literals)

DPLL(T) String Solvers



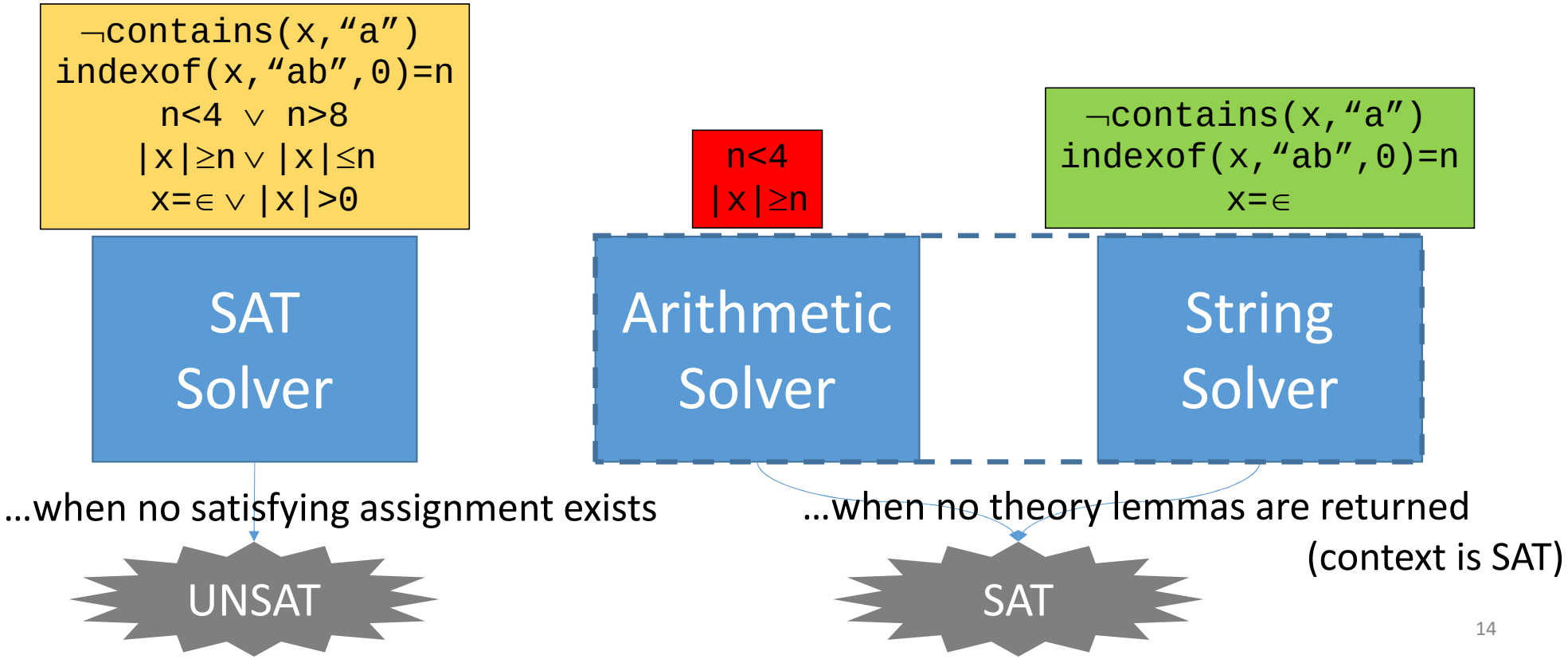
⇒ String and arithmetic solvers return theory lemmas to SAT solver

DPLL(T) String Solvers



⇒ …and repeat

DPLL(T) String Solvers



Properties of DPLL(T) String Solvers

- For *basic* constraints, DPLL(T) string solvers:
  - Can be used for "**sat**" and "**unsat**" answers
  - Are **incomplete** and/or **non-terminating** in general
- Expected, since *decidability is unknown*
  [Bjorner et al 2009,Ganesh et al 2011]
- Regardless, modern solvers are *efficient in practice*
  [Zheng et al 2013,Liang et al 2014,Abdulla et al 2015,Trinh et al 2016]

How do we handle Extended String Constraints?

`¬contains(x,“a”)`

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- Naively, by reduction to basic constraints + bounded ∀

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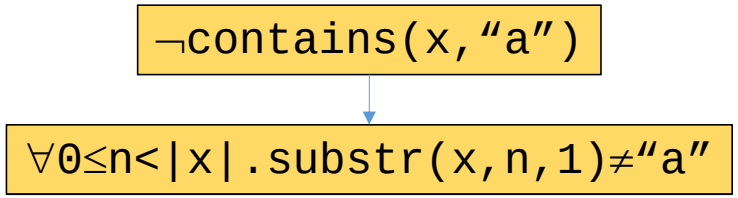
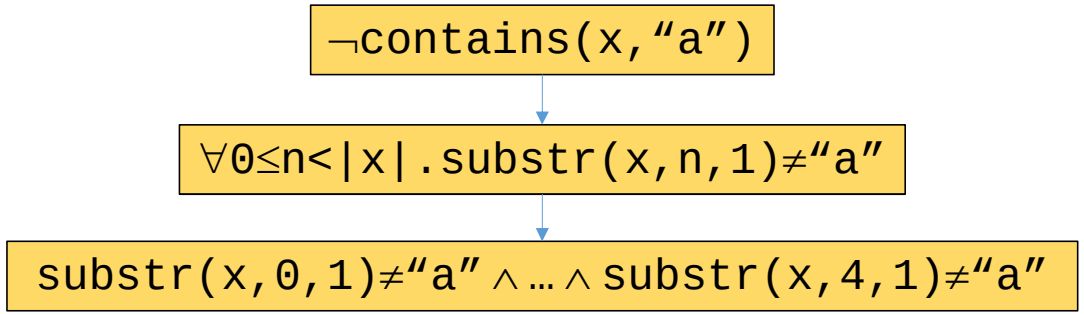


Expand `contains`

How do we handle Extended String Constraints?

- Naively, by reduction to basic constraints + bounded ∀



Expand `contains`

Assuming bound $|x| \leq 5$

How do we handle Extended String Constraints?

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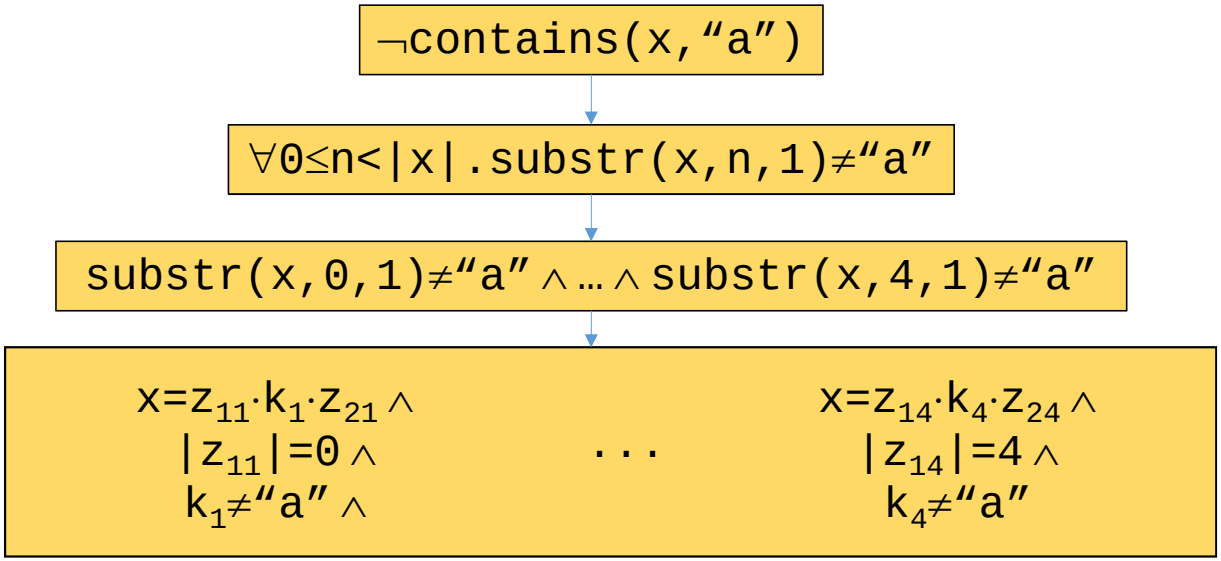

Expand `contains`

Assuming bound $|x| \leq 5$

Expand `substr`

How do we handle Extended String Constraints?

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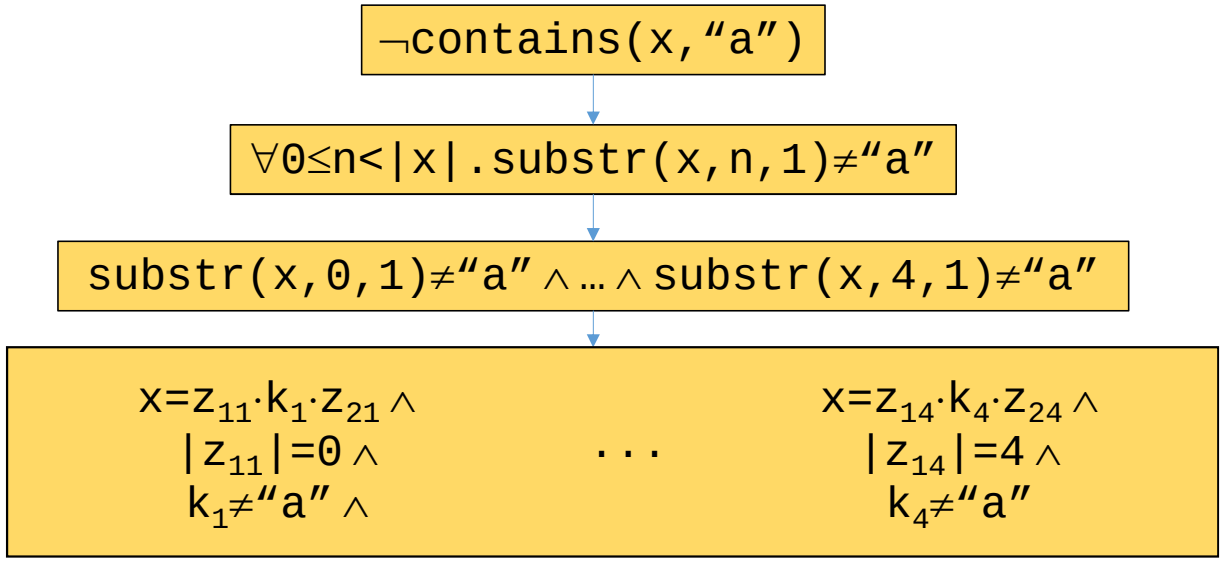


- Approach used by many current solvers
  [Bjorner et al 2009,Zheng et al 2013,Li et al 2013,Trinh et al 2014]

(Eager) Expansion of Extended Constraints

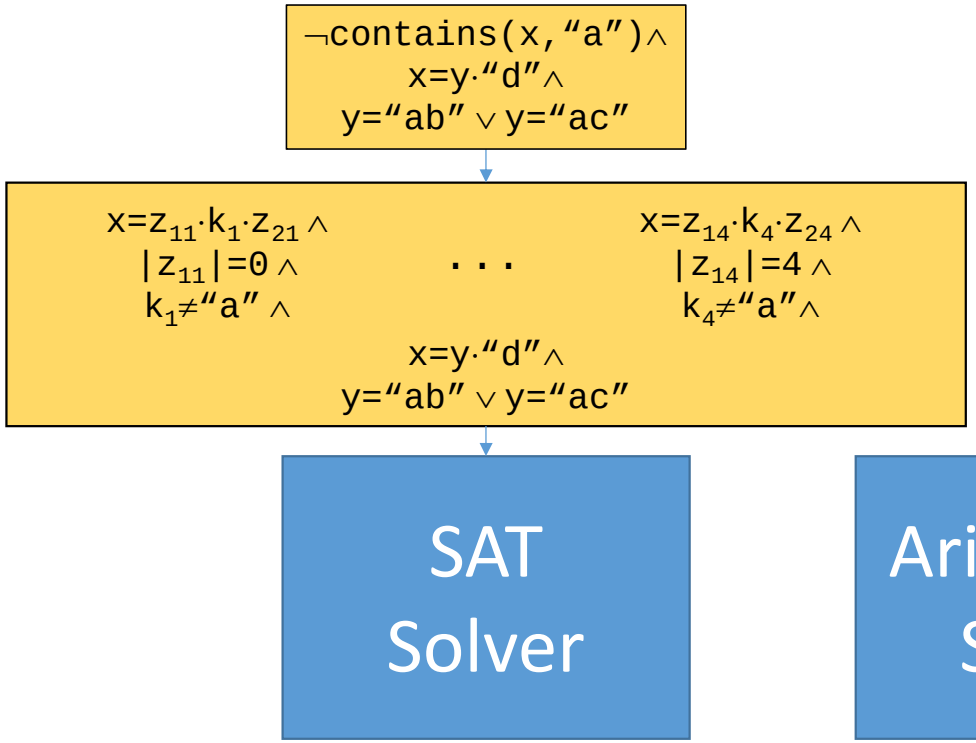
$\neg$contains(x,“a”)$\wedge$
x=y$\cdot$“d”$\wedge$
y=“ab”$\vee$y=“ac”

SAT Solver

Arithmetic Solver

String Solver

(Eager) Expansion of Extended Constraints



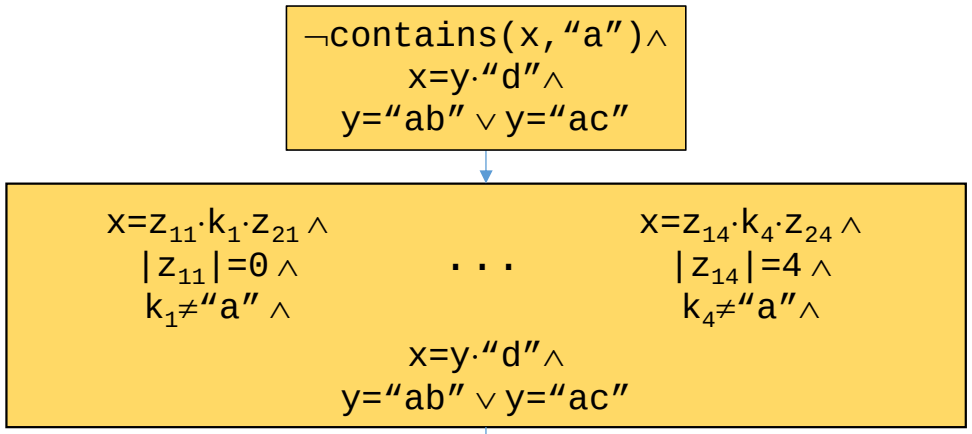
Expand and eliminate extended symbols

SAT Solver

Arithmetic Solver

String Solver

(Eager) Expansion of Extended Constraints



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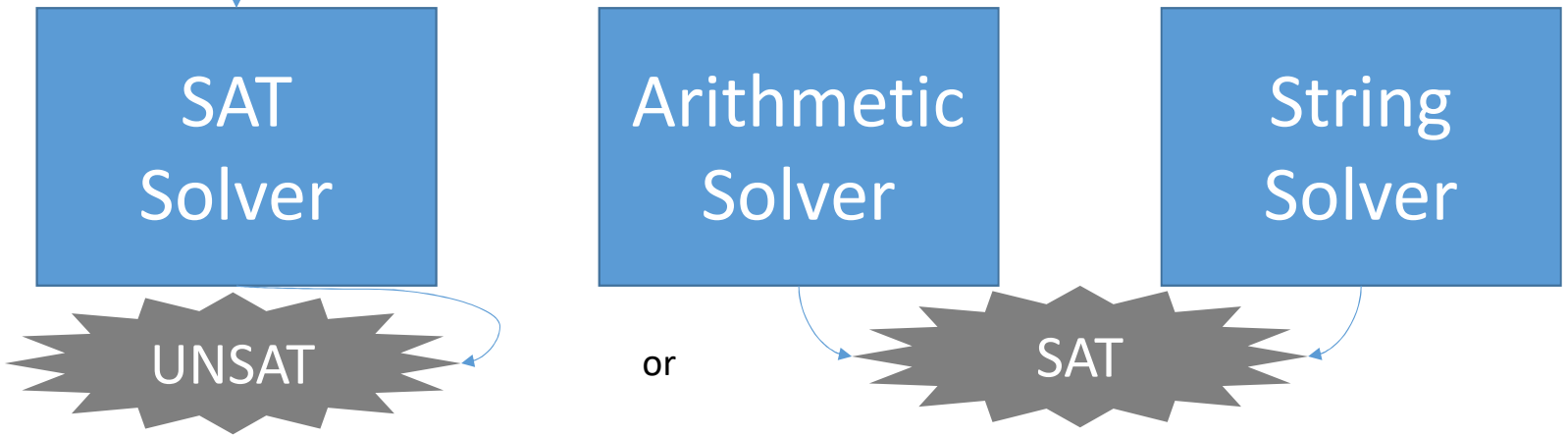

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SAT Solver

Arithmetic Solver

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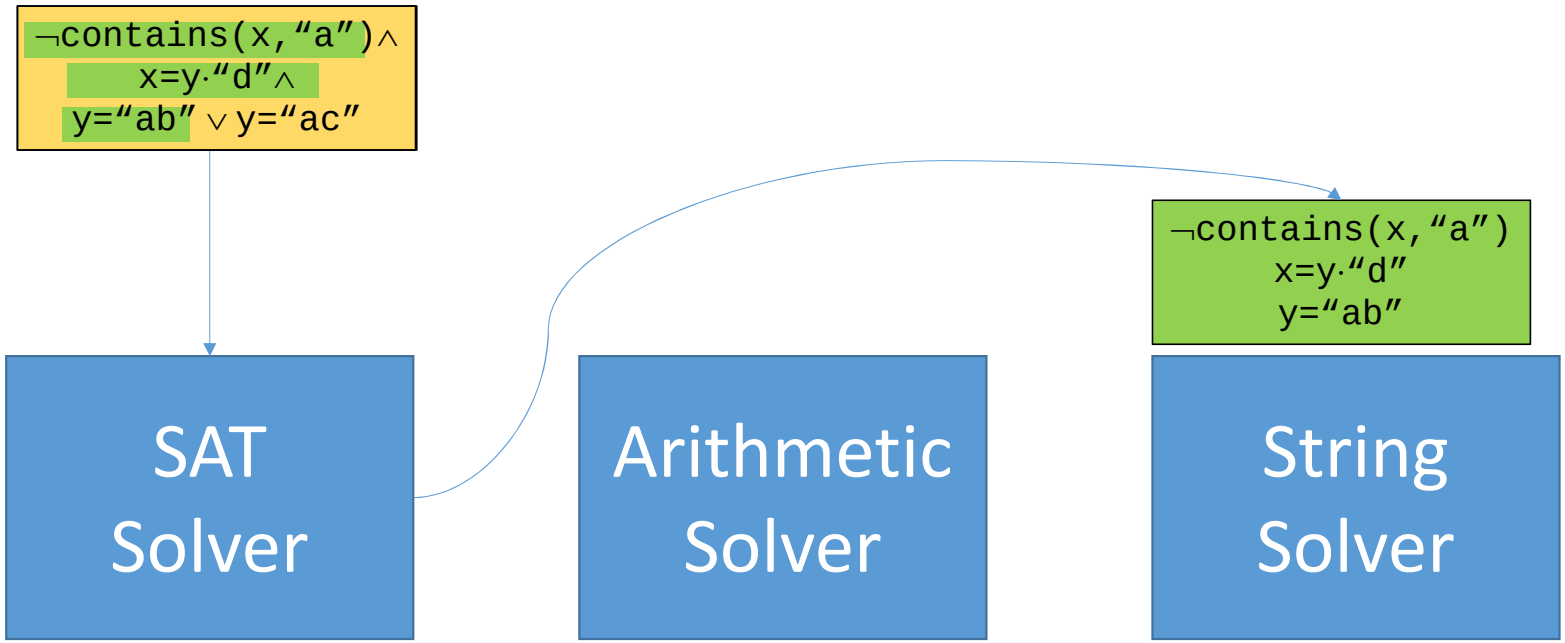

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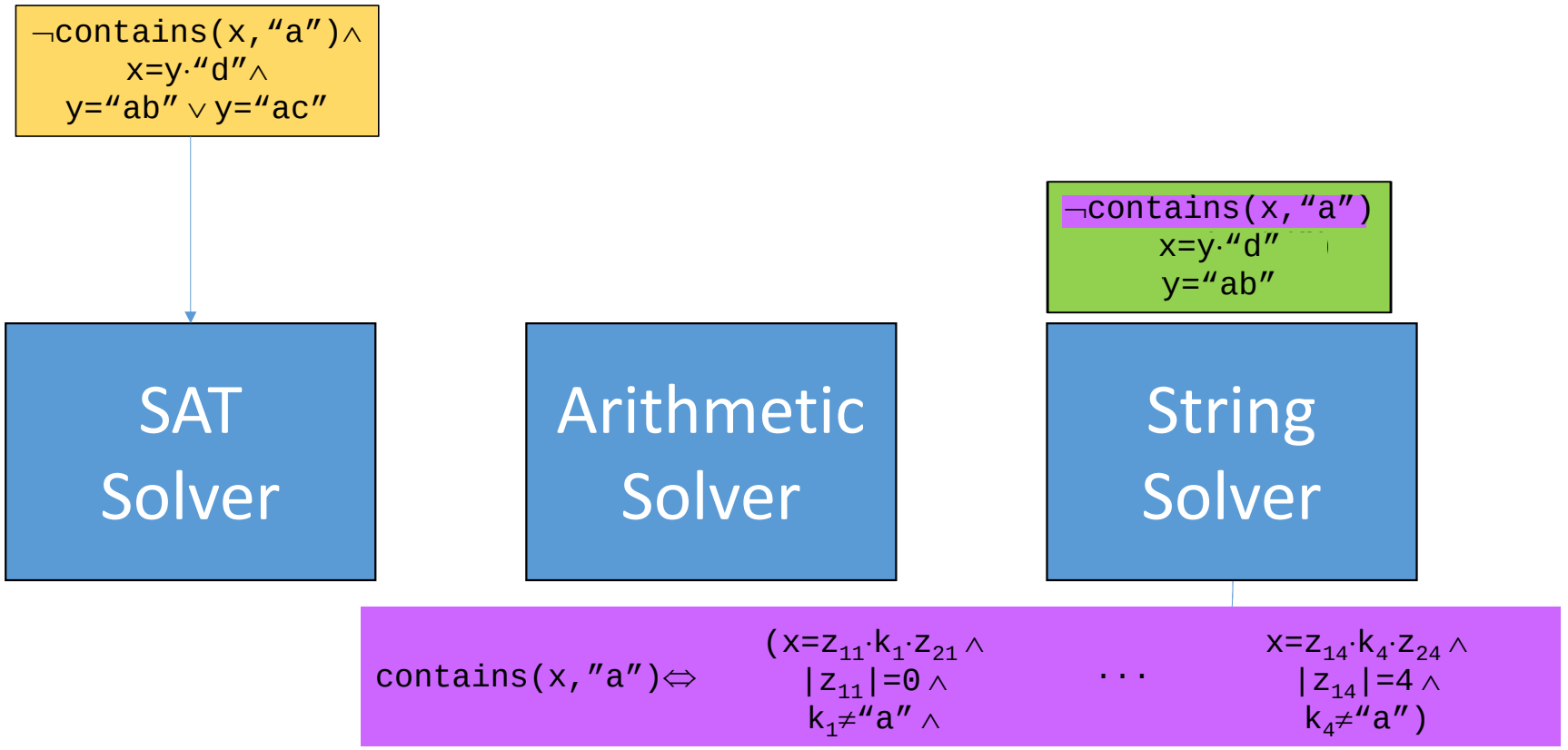

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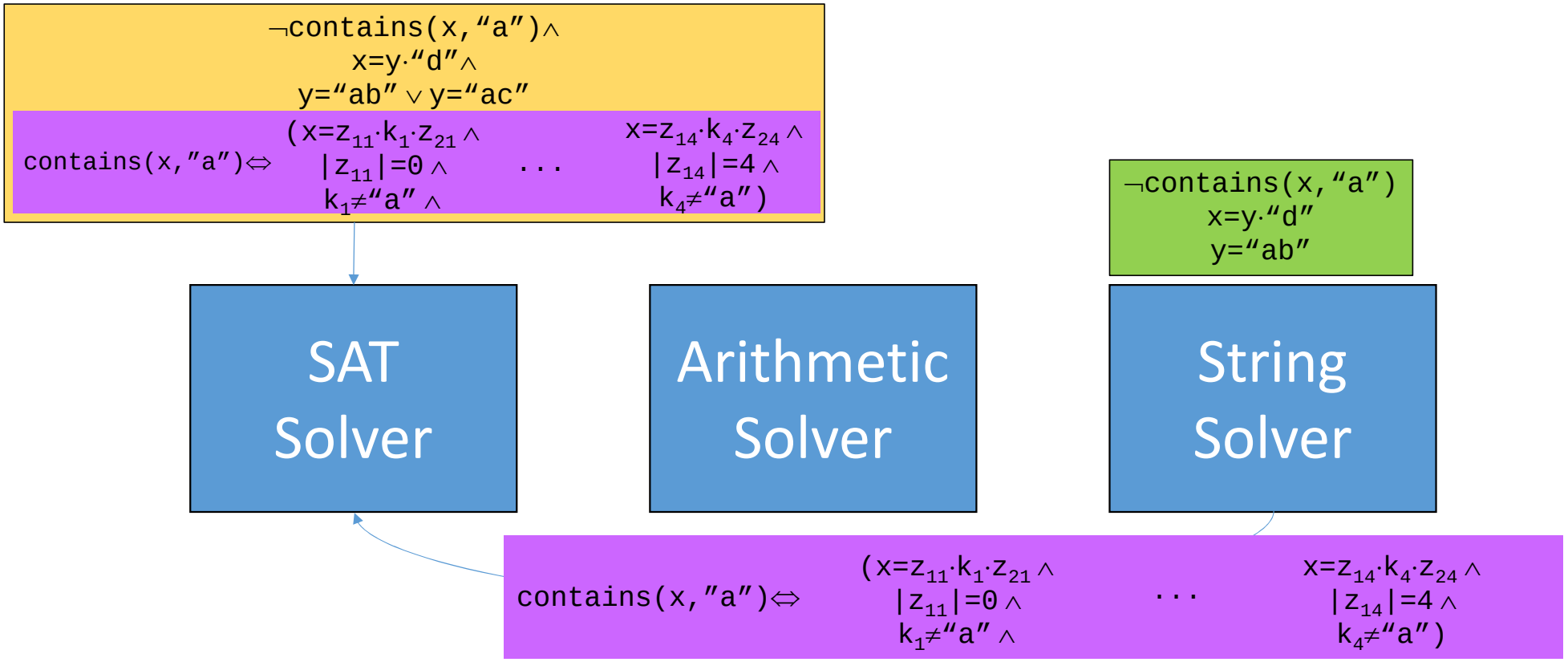

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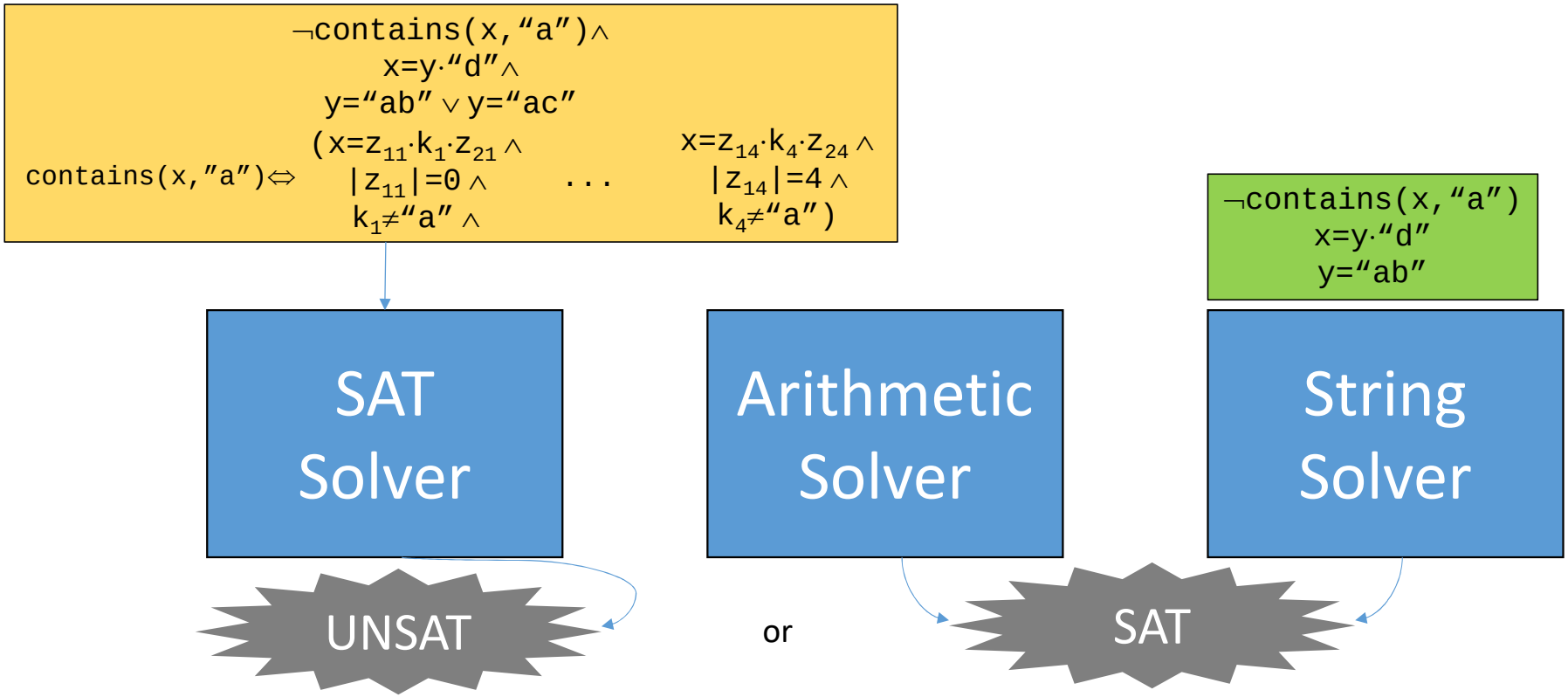

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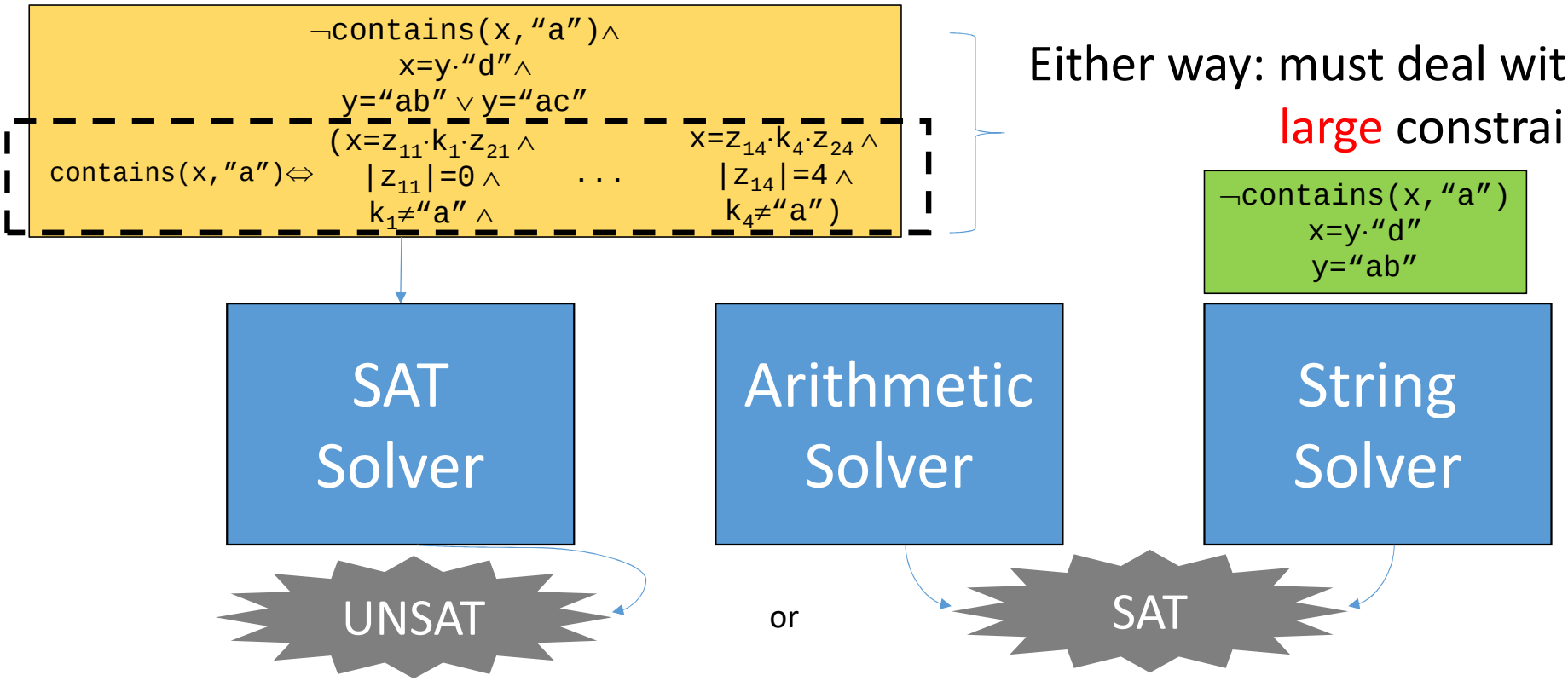

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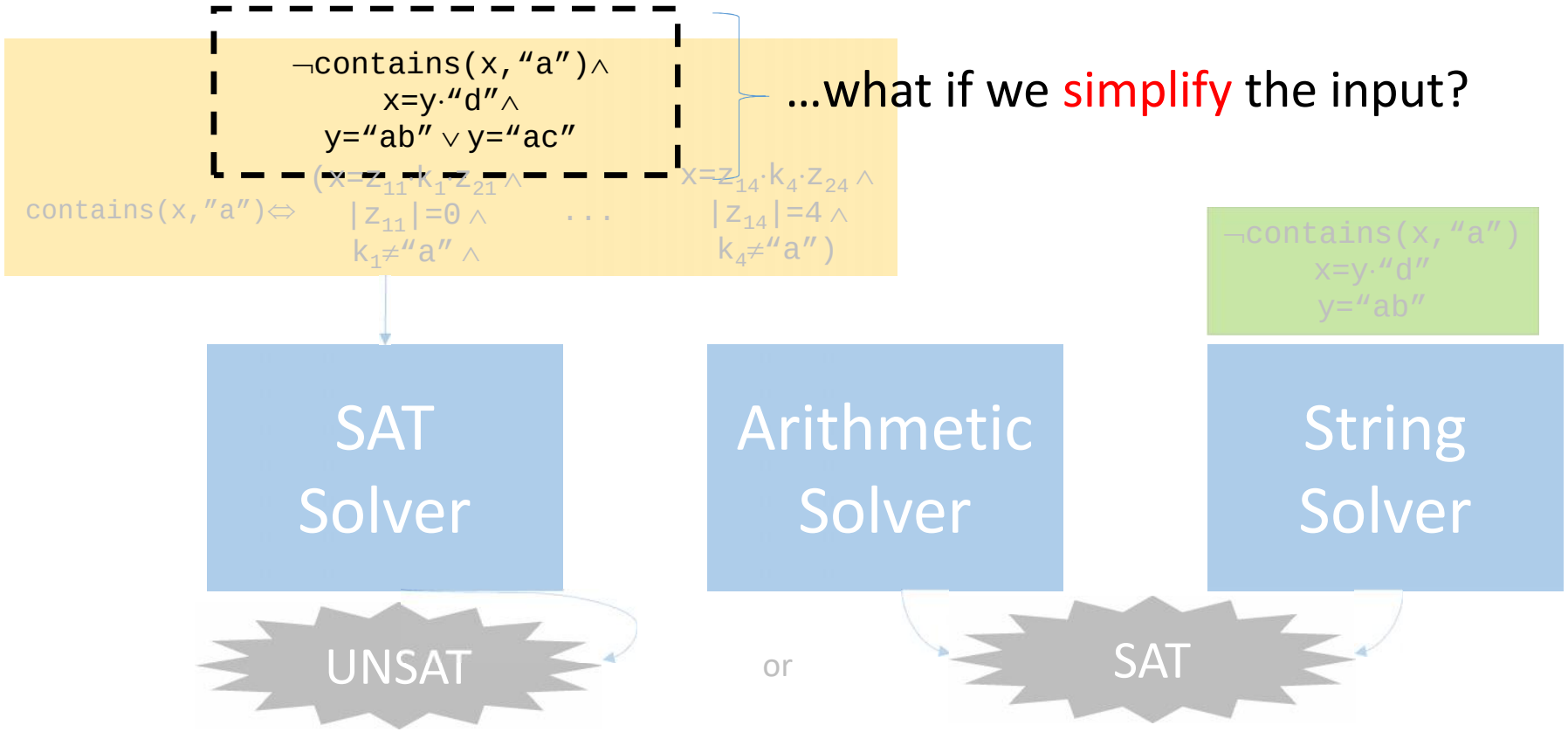

SMT Solvers + Simplification

- All SMT solvers implement *simplification* techniques
  (also called *normalization* or *rewrite* rules)

$$\neg\texttt{contains}(x,\text{“a”}) \wedge x = y \cdot \text{“d”} \wedge (y = \text{“ab”} \vee y = \text{“ac”})$$

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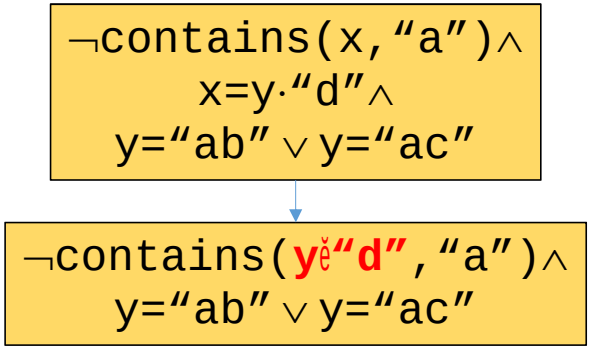

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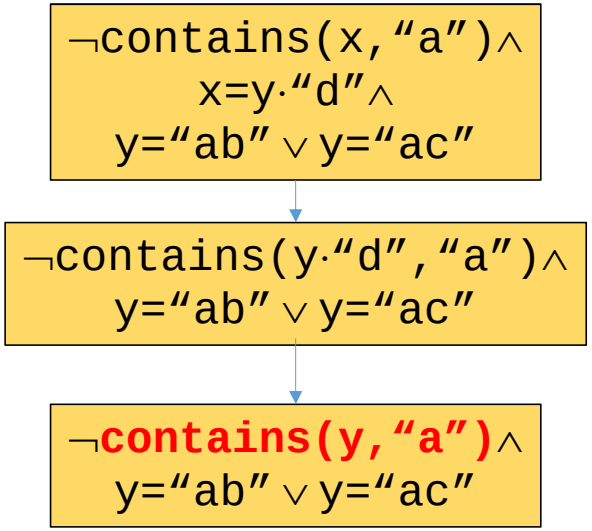

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since contains(y·“d”,”a”)⇔contains(y,“a”)

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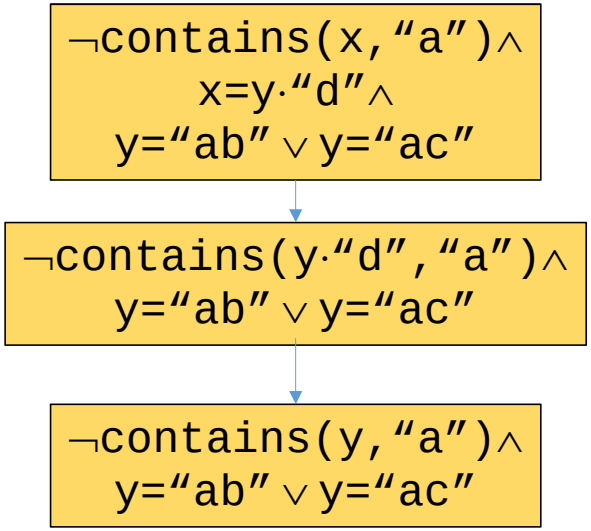

since x=y·”d”

since contains(y·“d”,”a”)⇔contains(y,“a”)

- Leads to smaller inputs, simpler procedures

(Lazy) Expansion + Simplification

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SAT Solver

Arithmetic Solver

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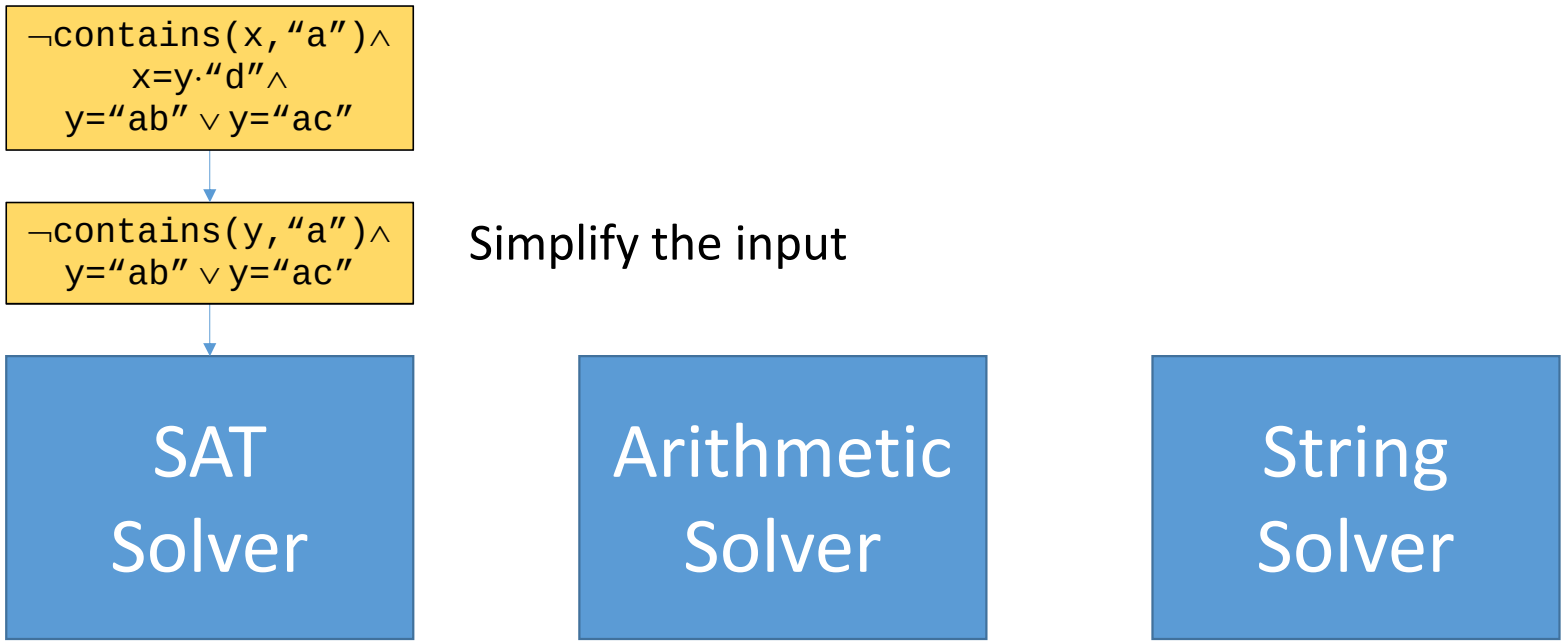
(Lazy) Expansion + Simplification



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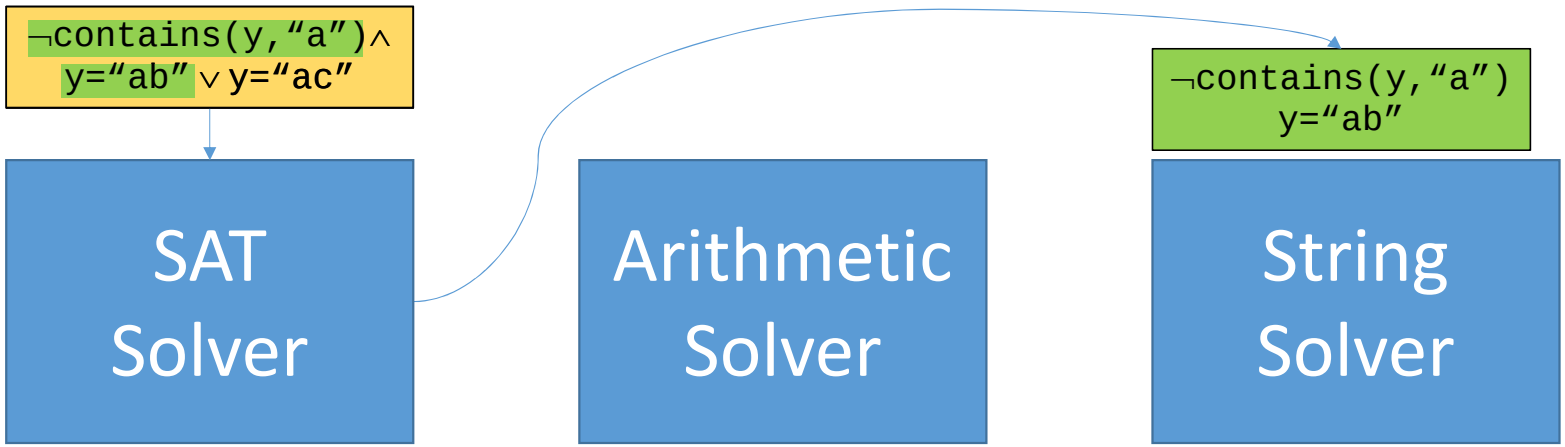

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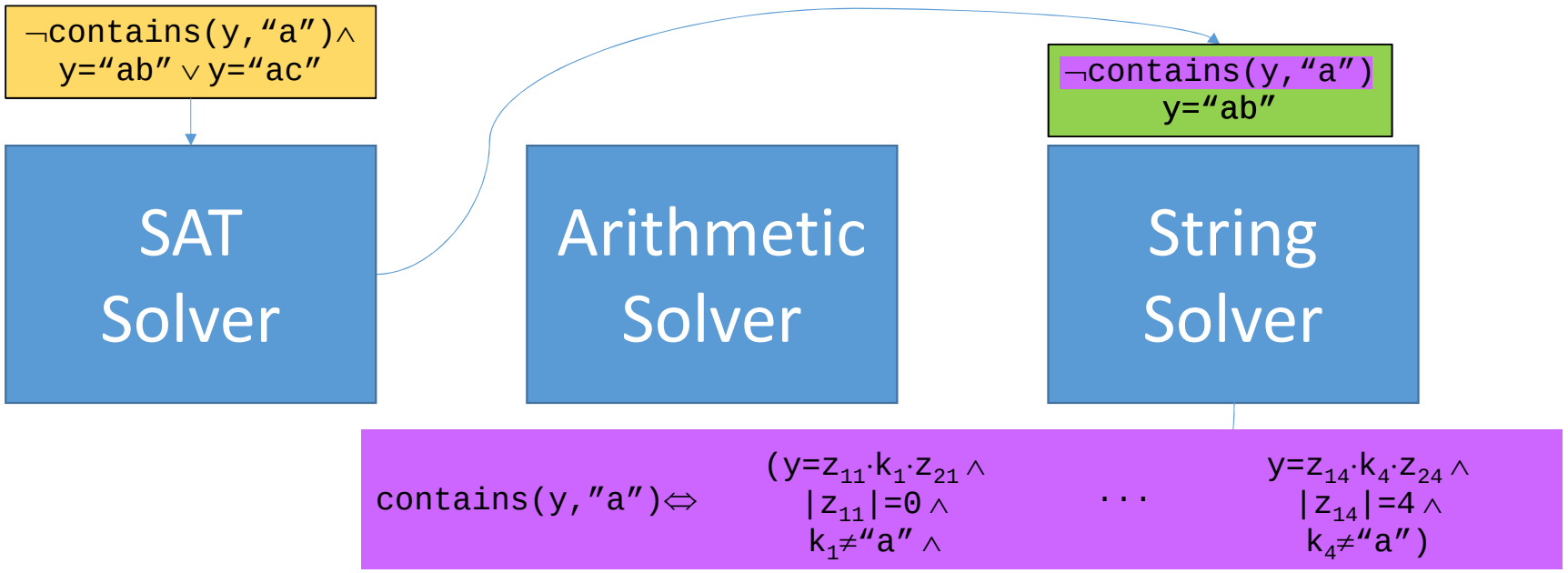

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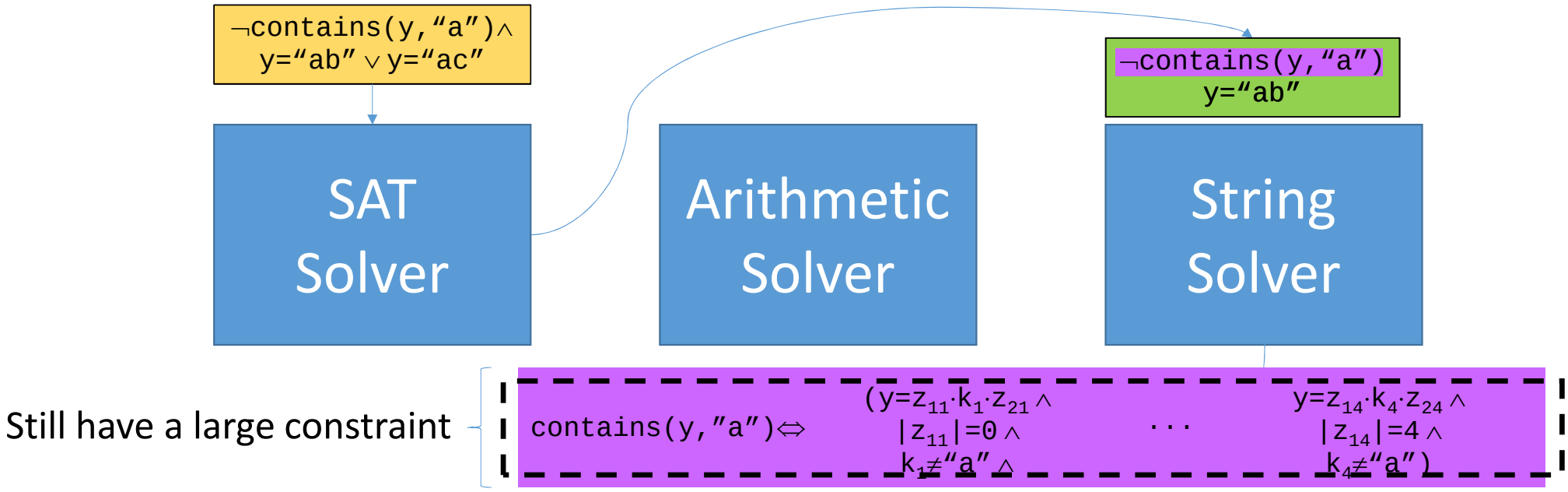
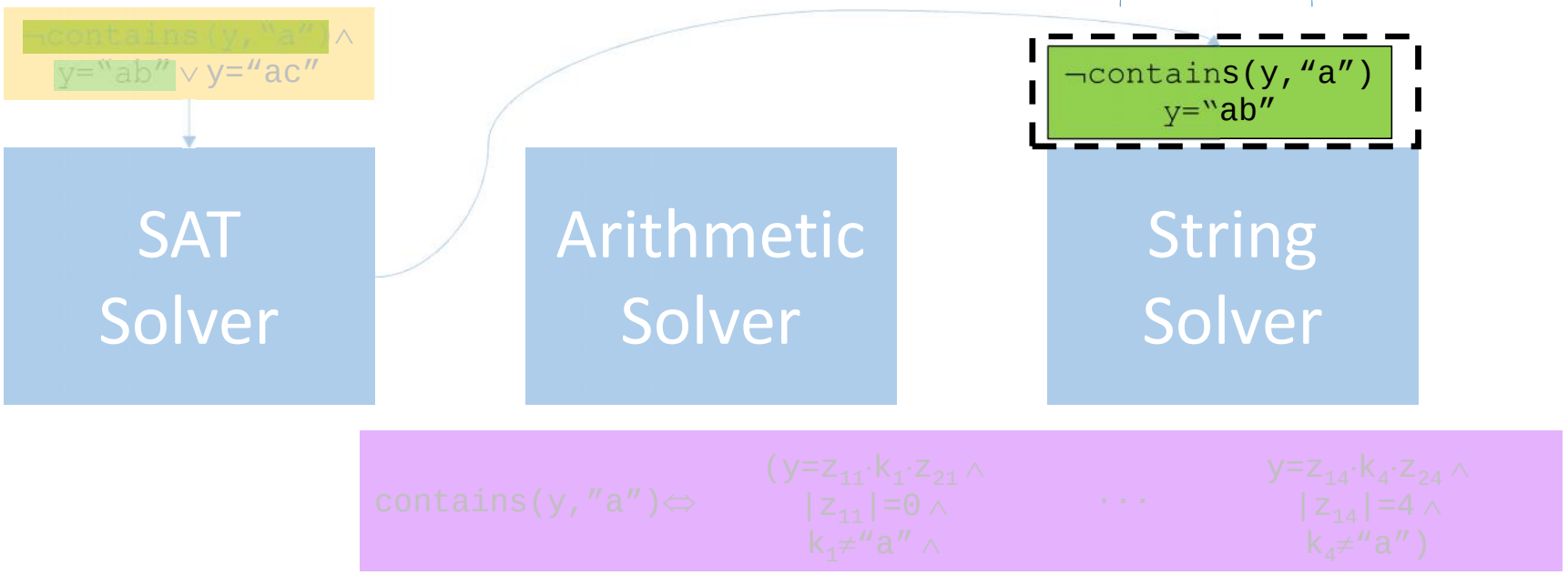


(Lazy) Expansion + Simplification

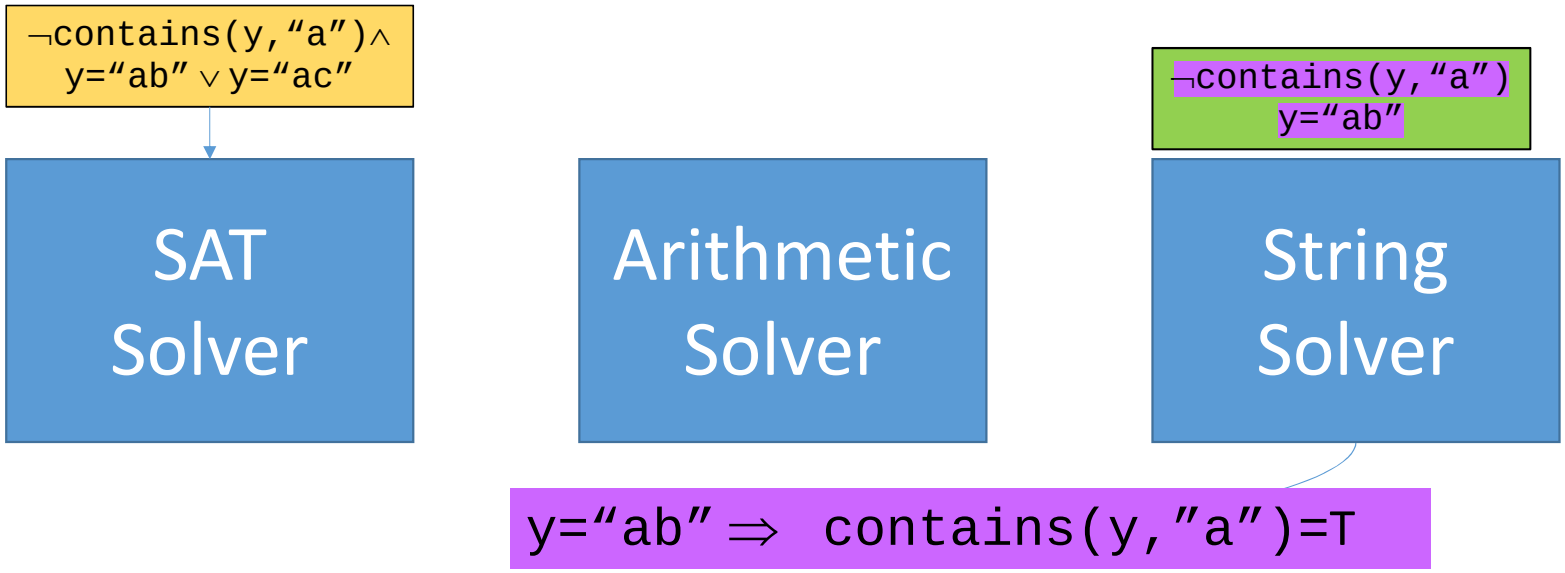
What if we simplify based on the context?



(Lazy) Expansion + Context-Dependent Simplification

Since `contains(y,"a")` is true when `y="ab"` …

(Lazy) Expansion + Context-Dependent Simplification



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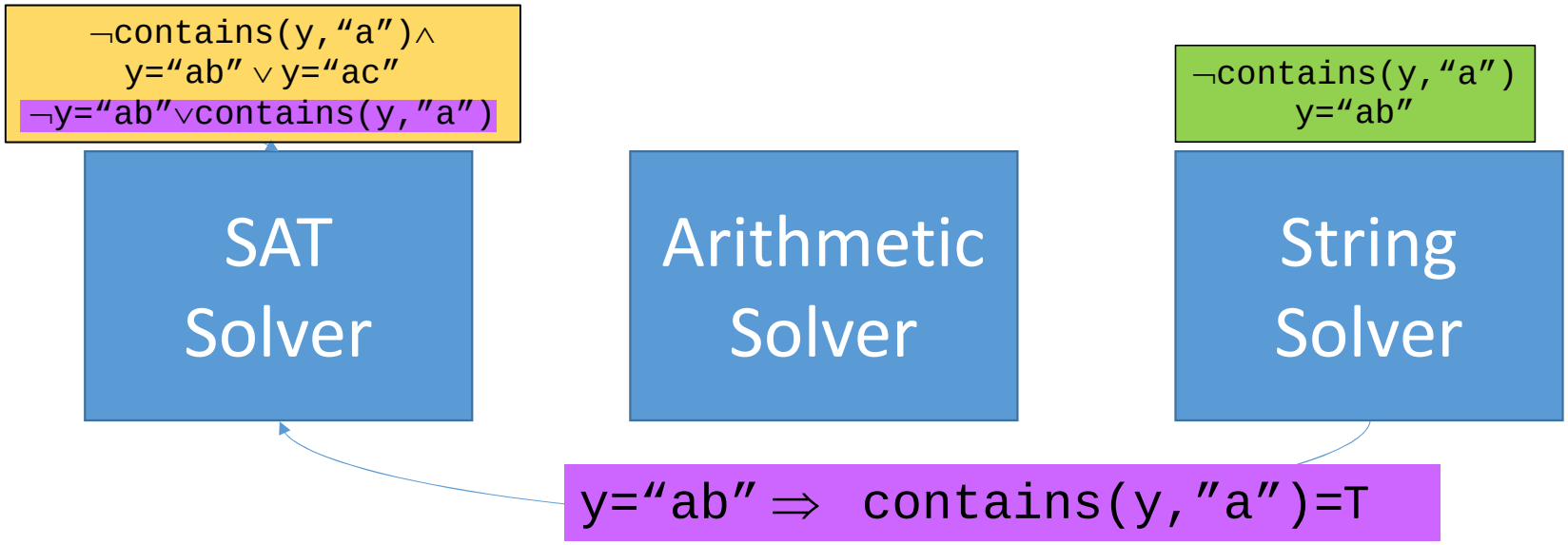

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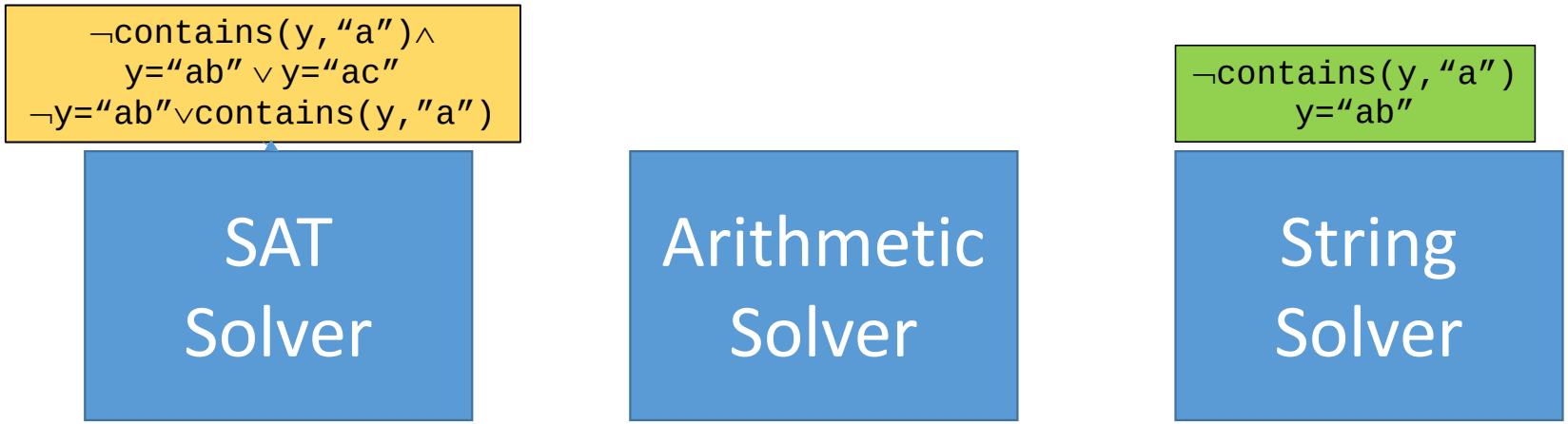

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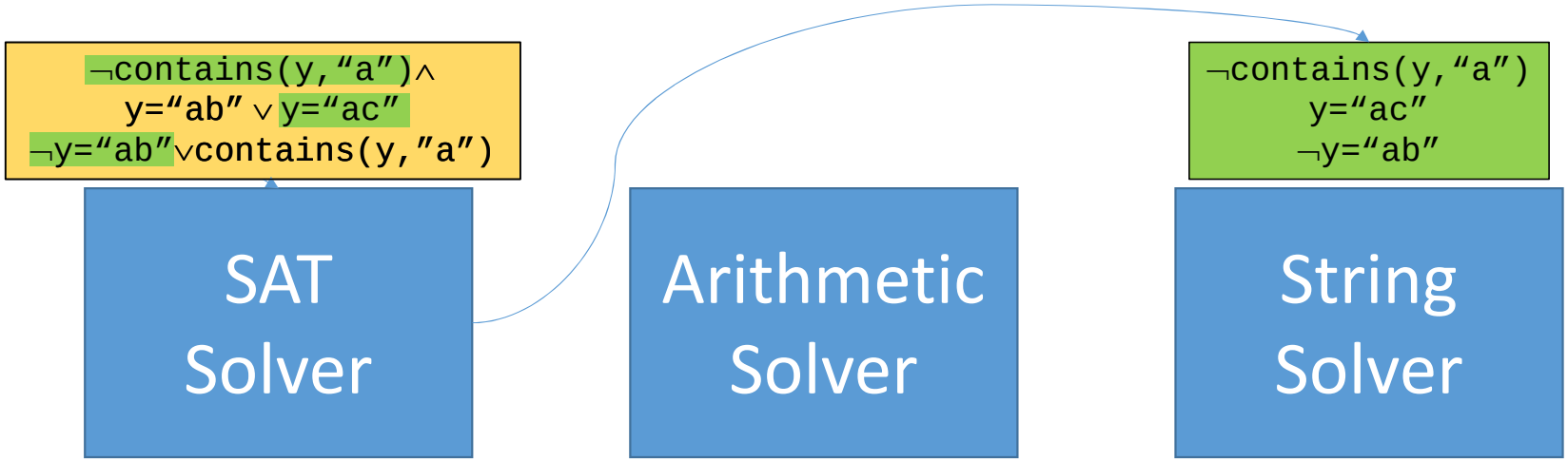

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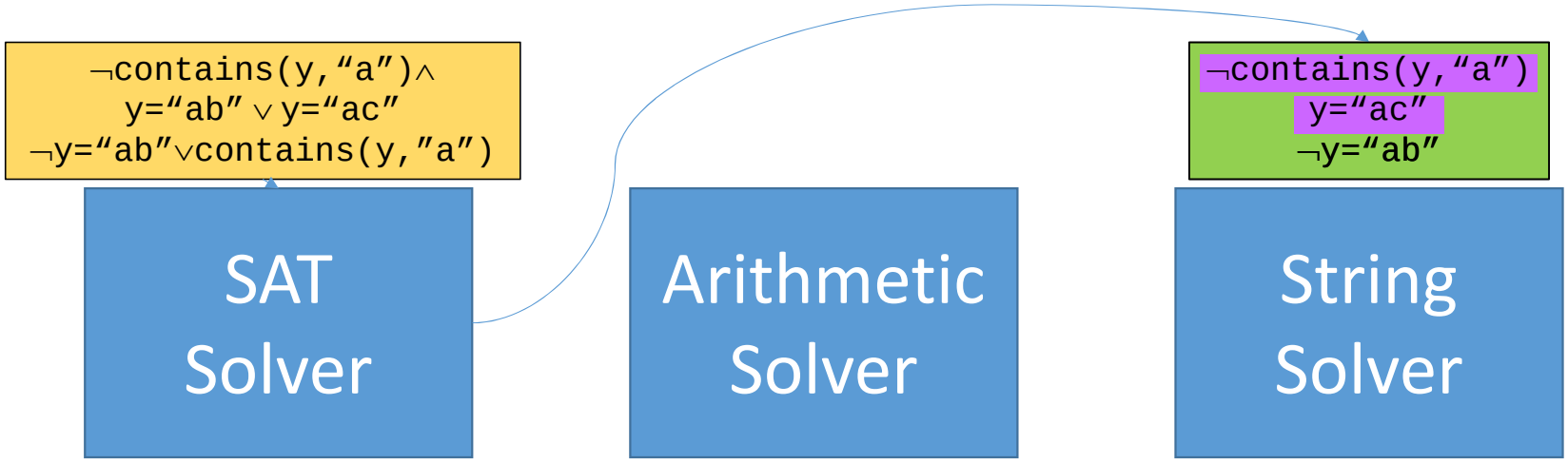
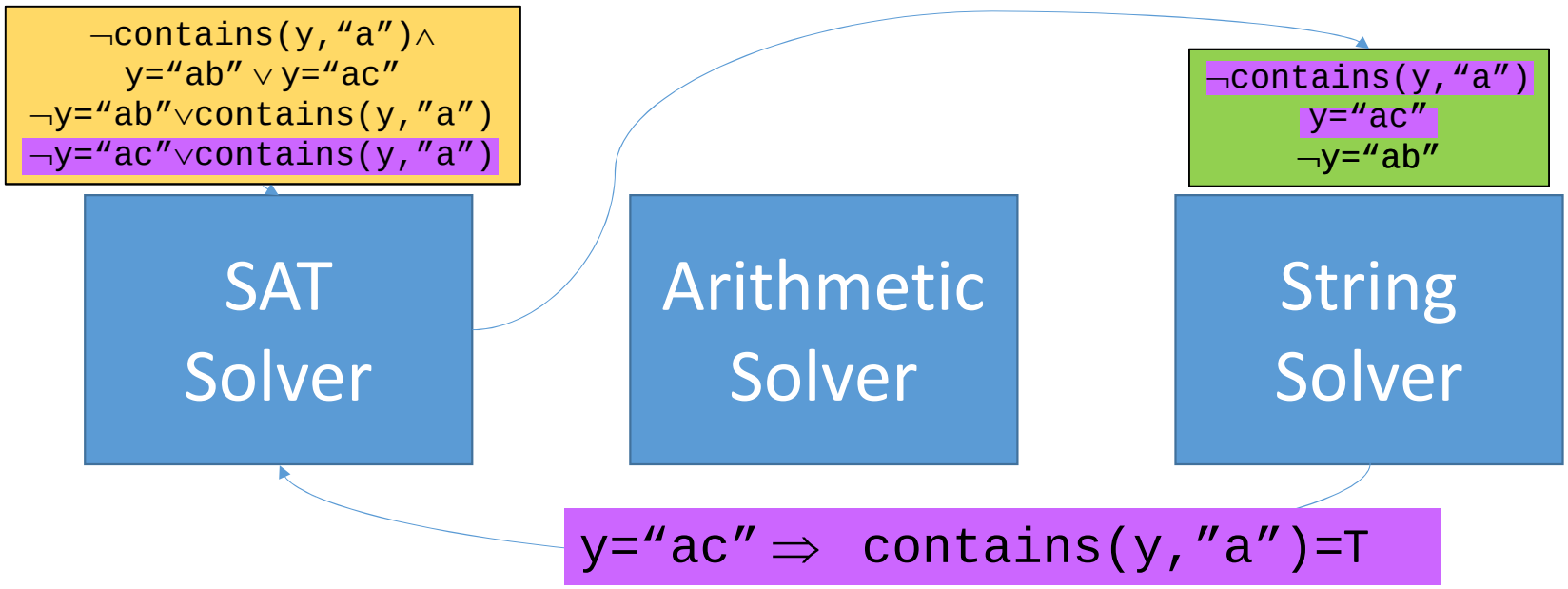


`contains(y,”a”)` is also true when `y=“ac”` …

(Lazy) Expansion + Context-Dependent Simplification



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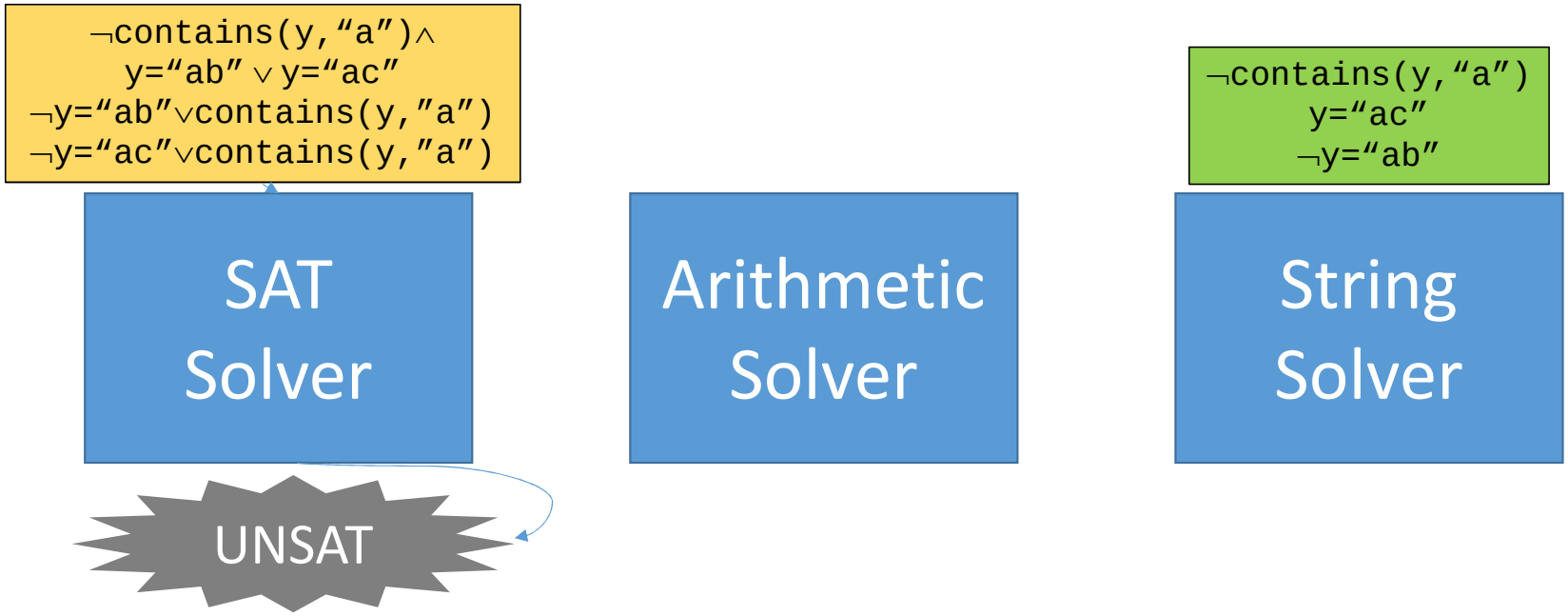

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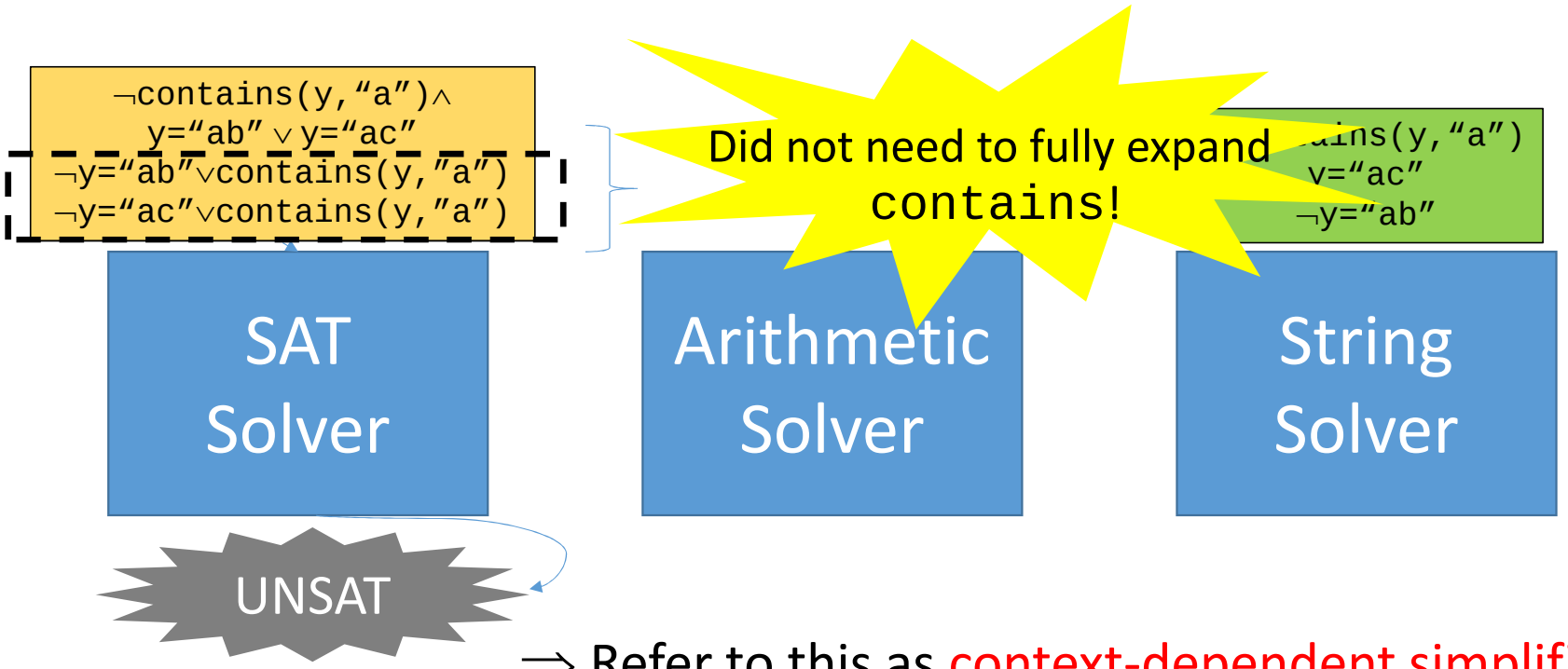
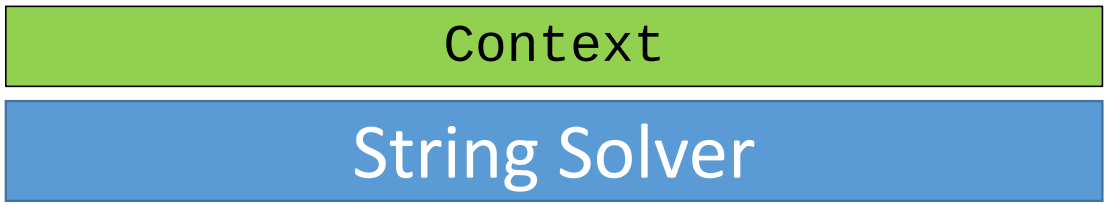
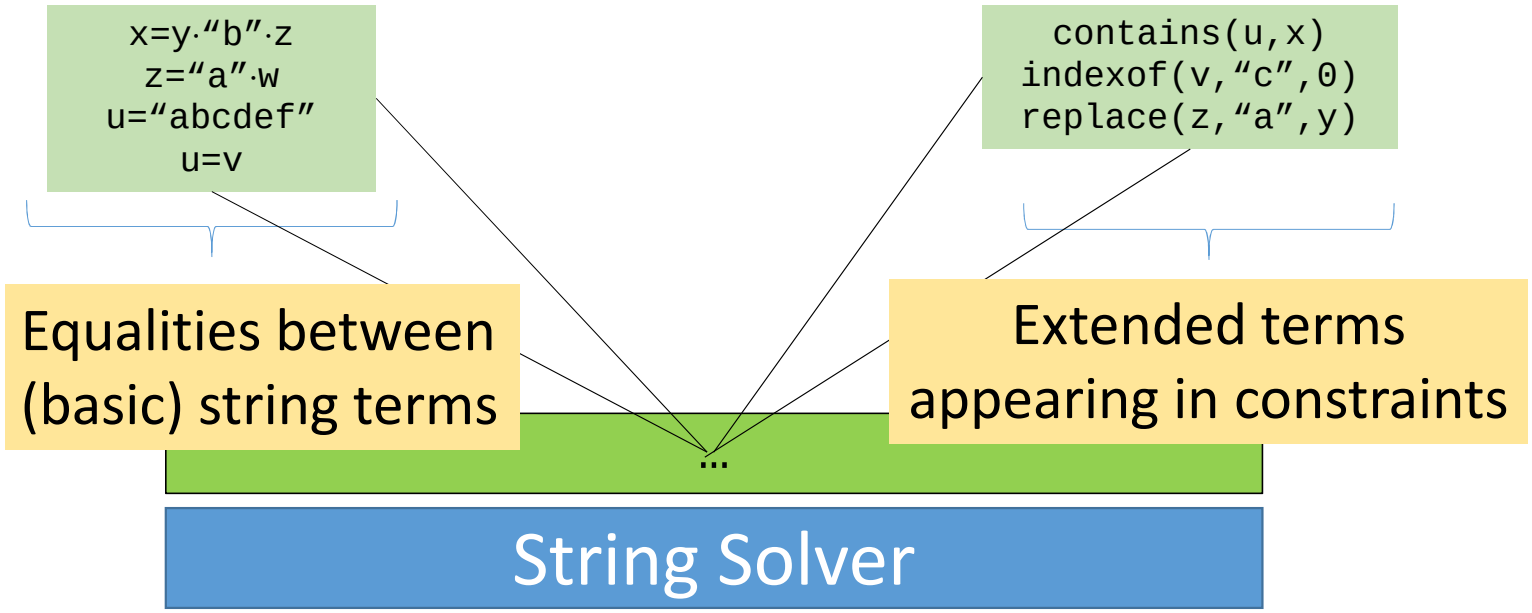


⇒ Refer to this as context-dependent simplification

Context-Dependent Simplification



Context-Dependent Simplification

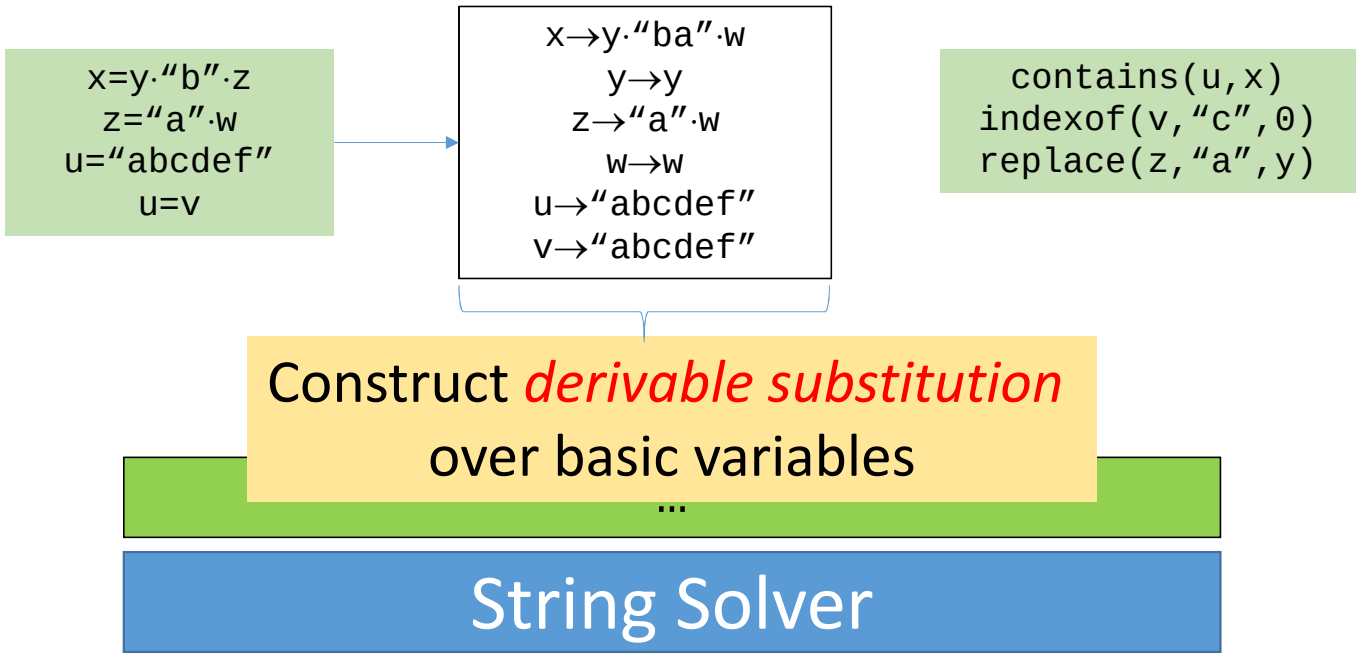


Context-Dependent Simplification

```
x=y·“b”·z
z=“a”·w
u=“abcdef”
u=v
```

```
contains(u,x)
indexof(v,“c”,0)
replace(z,“a”,y)
```

Context-Dependent Simplification



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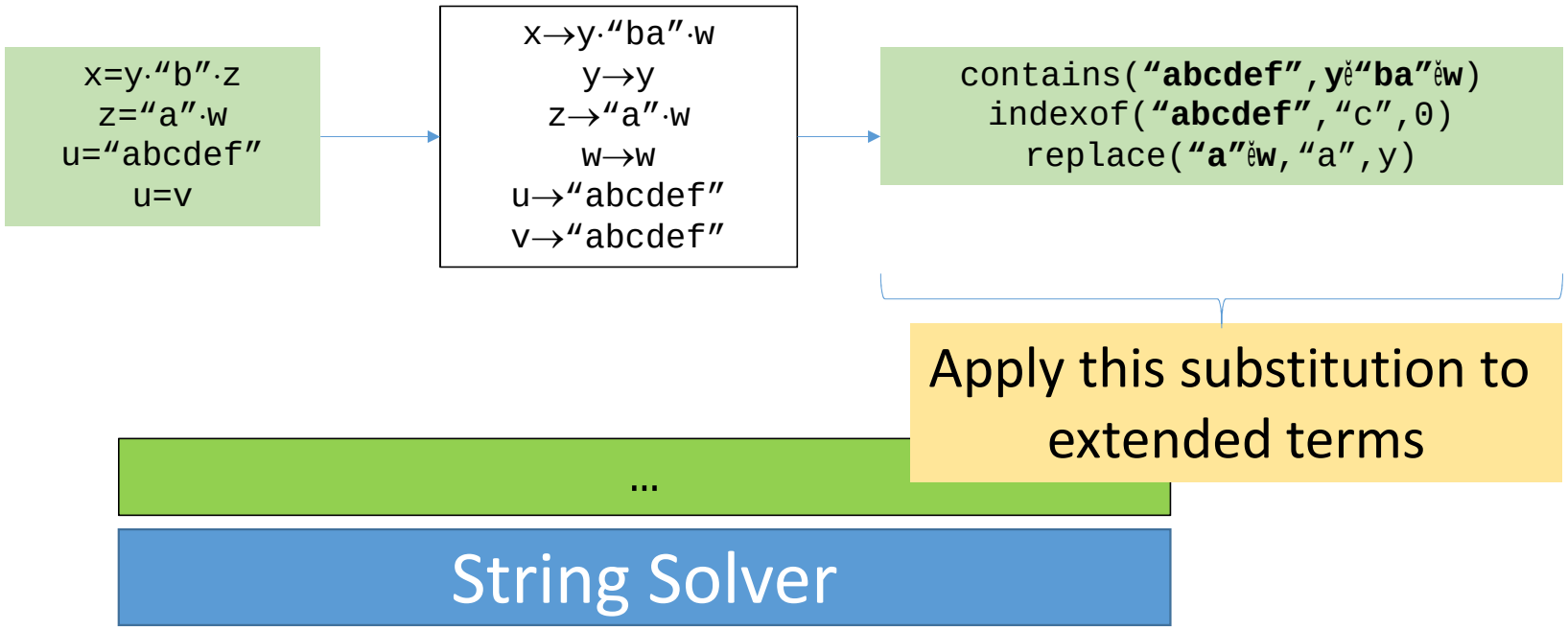

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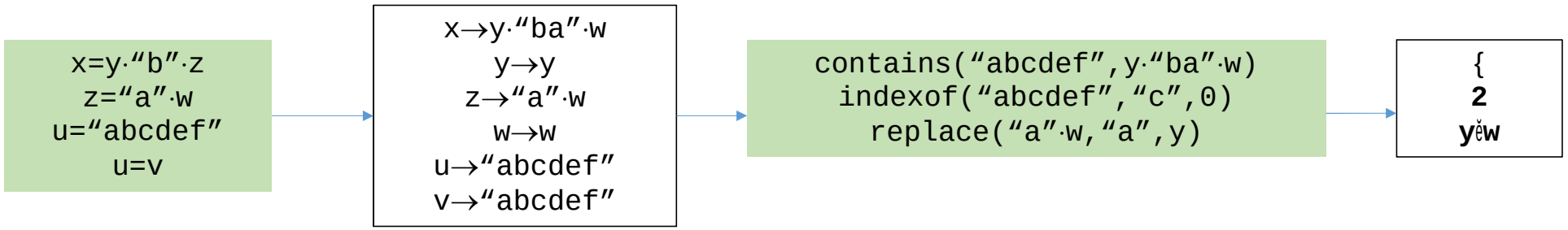
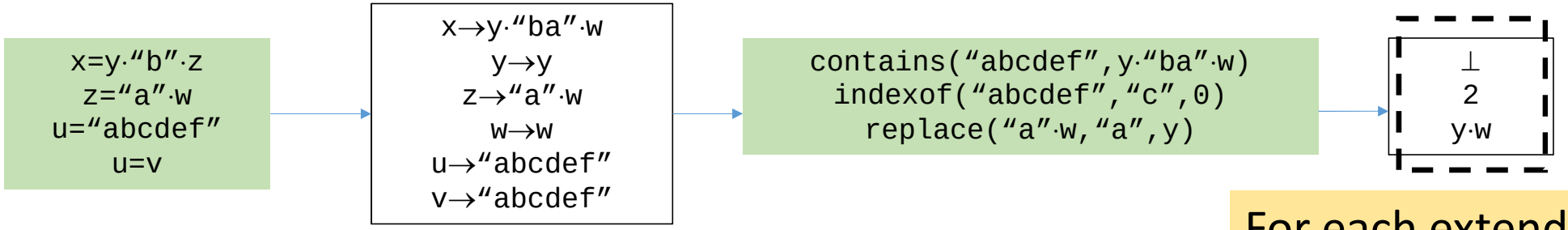


Simplify results

Context-Dependent Simplification

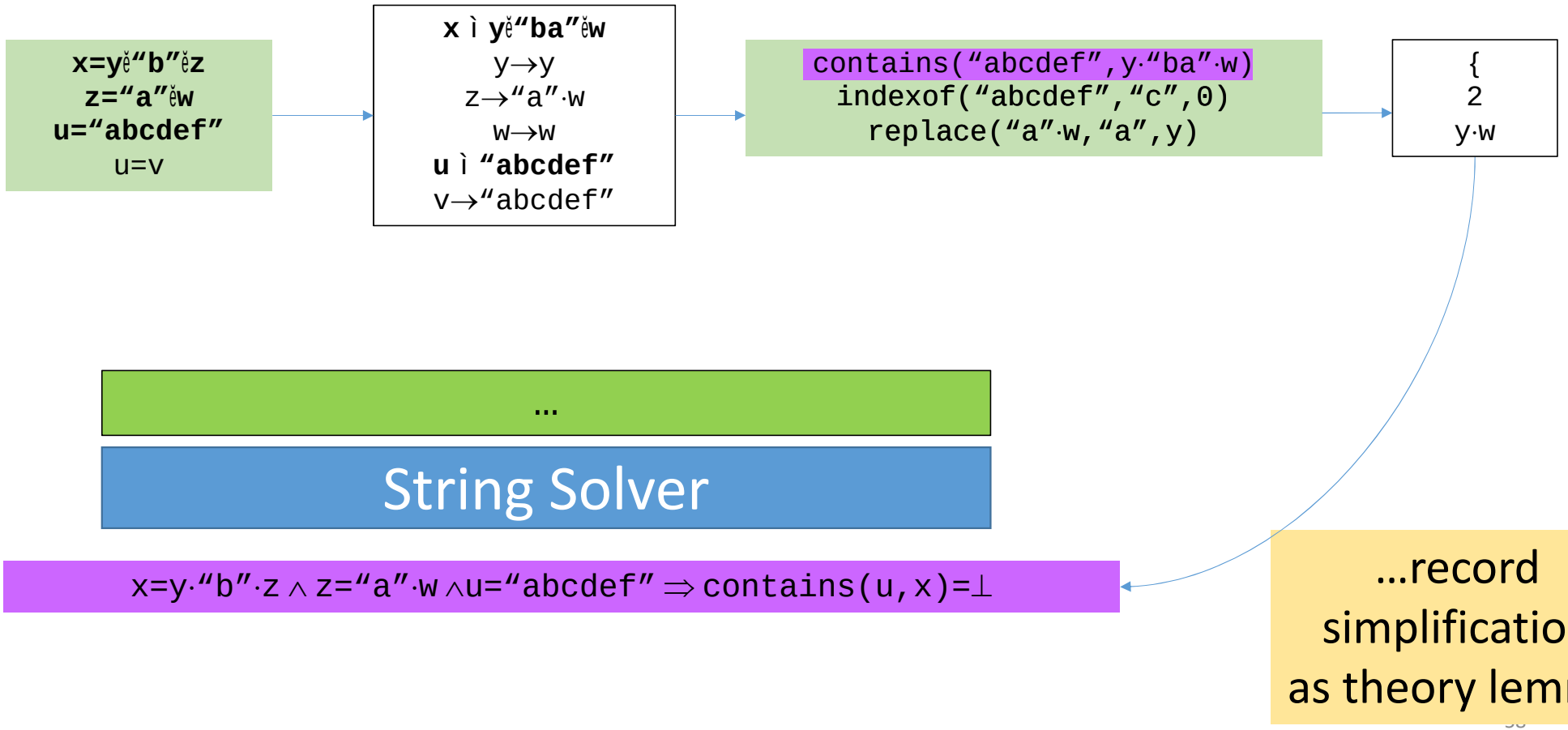


For each extended term that was simplified to a basic one…

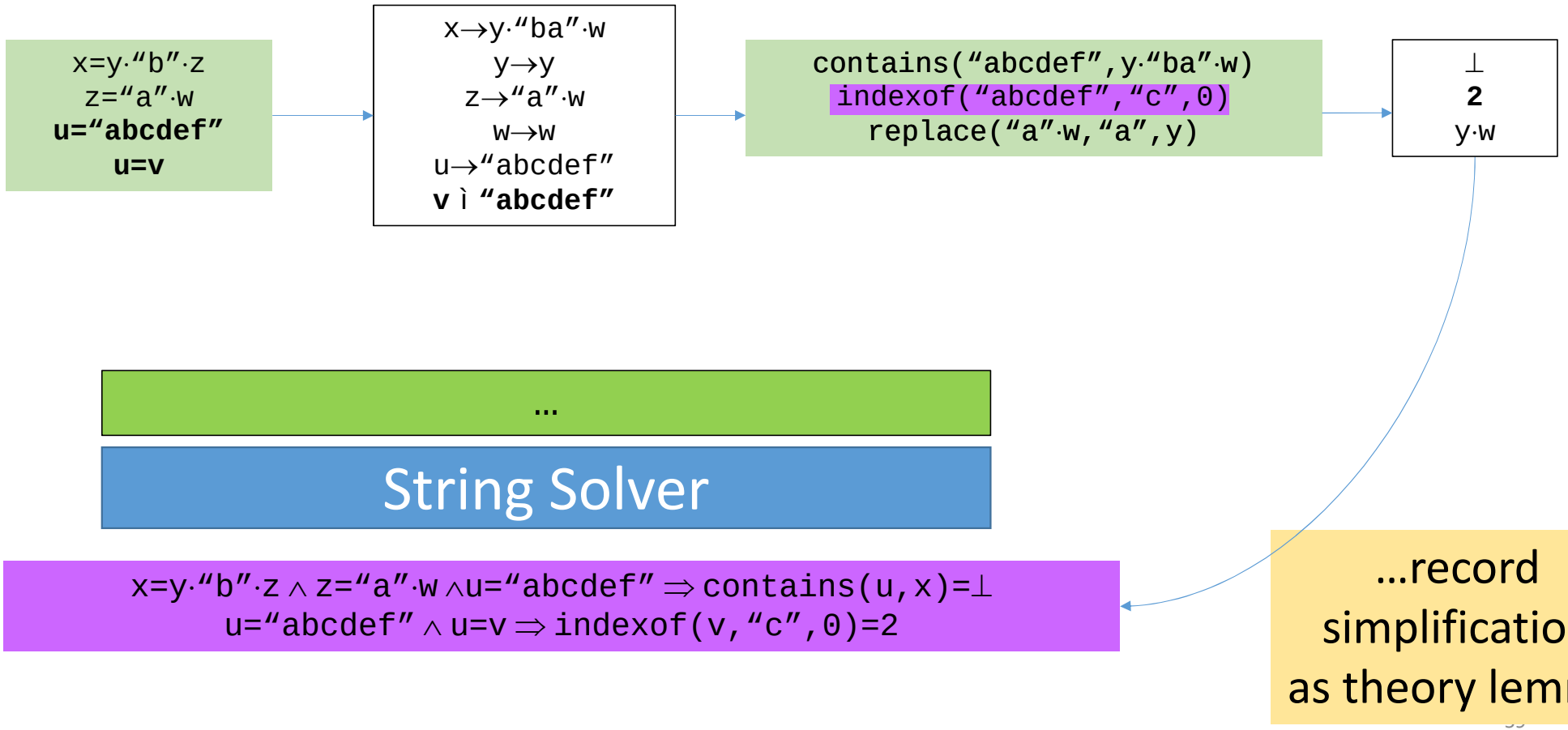
…

String Solver

Context-Dependent Simplification



Context-Dependent Simplification



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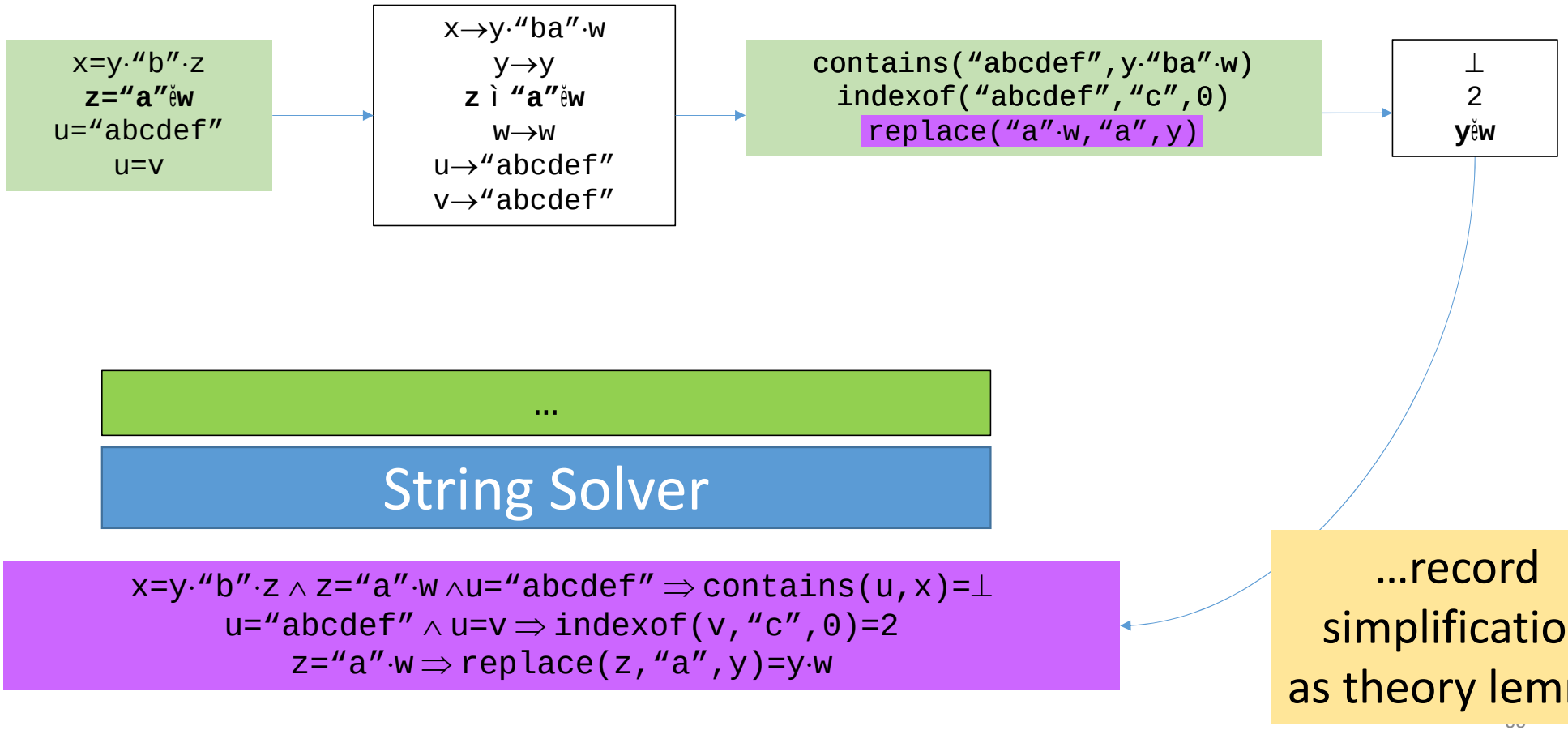
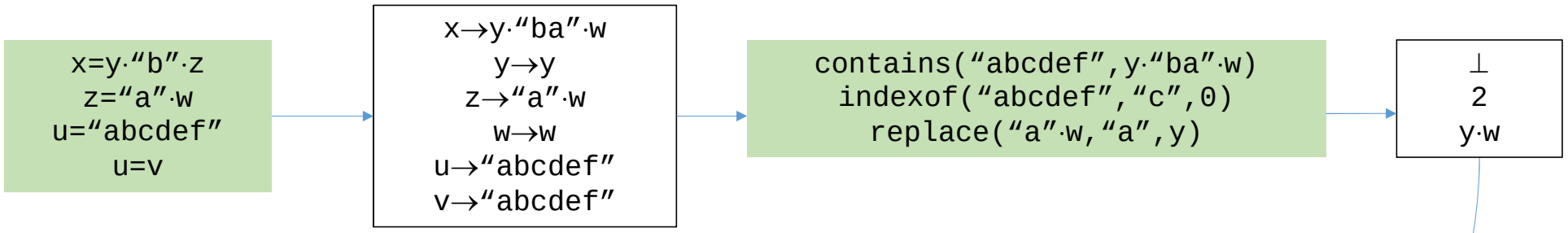


Context-Dependent Simplification



…

String Solver

$x=y\cdot\text{“b”}\cdot z \wedge z=\text{“a”}\cdot w \wedge u=\text{“abcdef”} \Rightarrow \texttt{contains}(u,x)=\bot$
$u=\text{“abcdef”} \wedge u=v \Rightarrow \texttt{indexof}(v,\text{“c”},0)=2$
$z=\text{“a”}\cdot w \Rightarrow \texttt{replace}(z,\text{“a”},y)=y\cdot w$

In our dataset, inference of this form is possible in 98% of contexts

Simplification Rules for Strings

- Unlike arithmetic:

`x+x+7*y=y-4` → `2*x+6*y+4=0`

…simplification rules for strings are highly non-trivial:

`substr(x·“abcd”,1+len(x),2)`	`“bc”`
`contains(“abcde”,“b”·x·“a”)`	⊥
`contains(x·“ac”·y,“b”)`	`contains(x,“b”)∨contains(y,“b”)`
`indexof(“abc”·x,“a”·x,1)`	`-1`
`replace(“a”·x,“b”,y)`	`con(“a”,replace(x,“b”,y))`

- Implemented in 3000+ lines of C++ code

Theoretical Contribution

- Approach described as a rule-based *calculus,* e.g.:

$$\dots$$

$$\text{Ext-Simplify}\ \frac{x \approx t \in \mathsf{X} \qquad \mathsf{E} \models \boldsymbol{y} \approx \boldsymbol{s} \qquad (t\{\boldsymbol{y} \mapsto \boldsymbol{s}\})\downarrow \text{ is a } \Sigma_{\mathsf{AS}}\text{-term}}{\mathsf{G} := \mathsf{G}, \lfloor x \approx (t\{\boldsymbol{y} \mapsto \boldsymbol{s}\})\downarrow \rfloor \qquad \mathsf{X} := \mathsf{X} \backslash \{x \approx t\}}$$

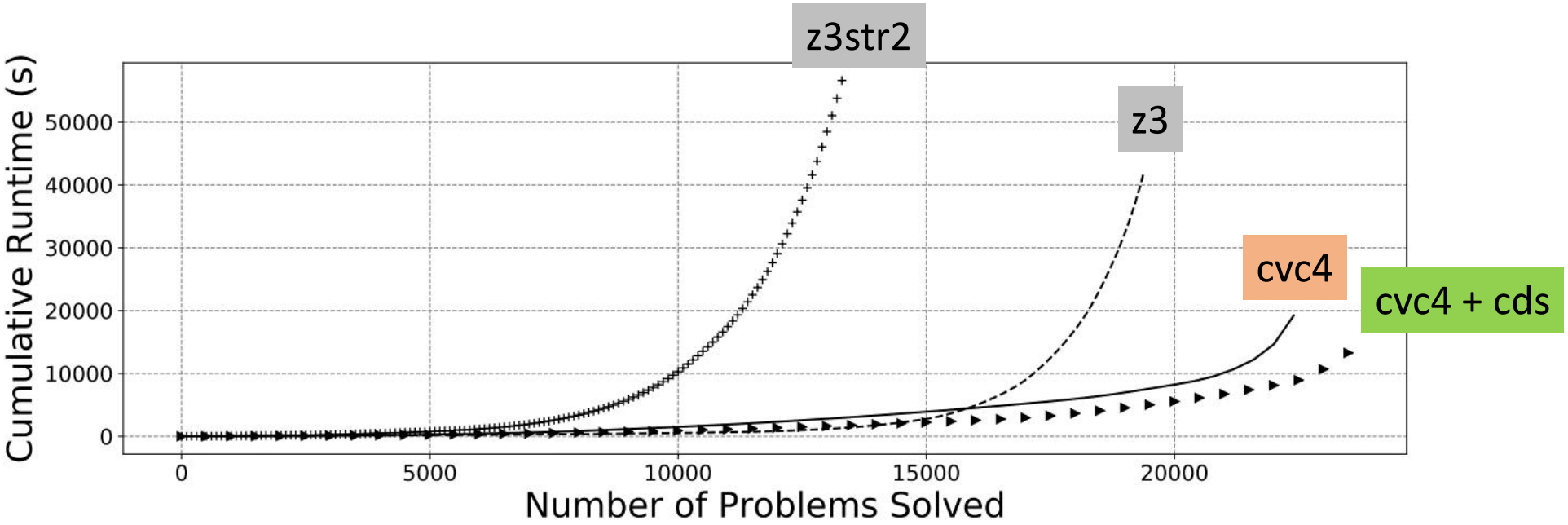
$$\dots$$

- Calculus is:
  - **Refutation-sound**
  - **Model-sound**
  - **Not terminating** in general (decidability is still unknown)

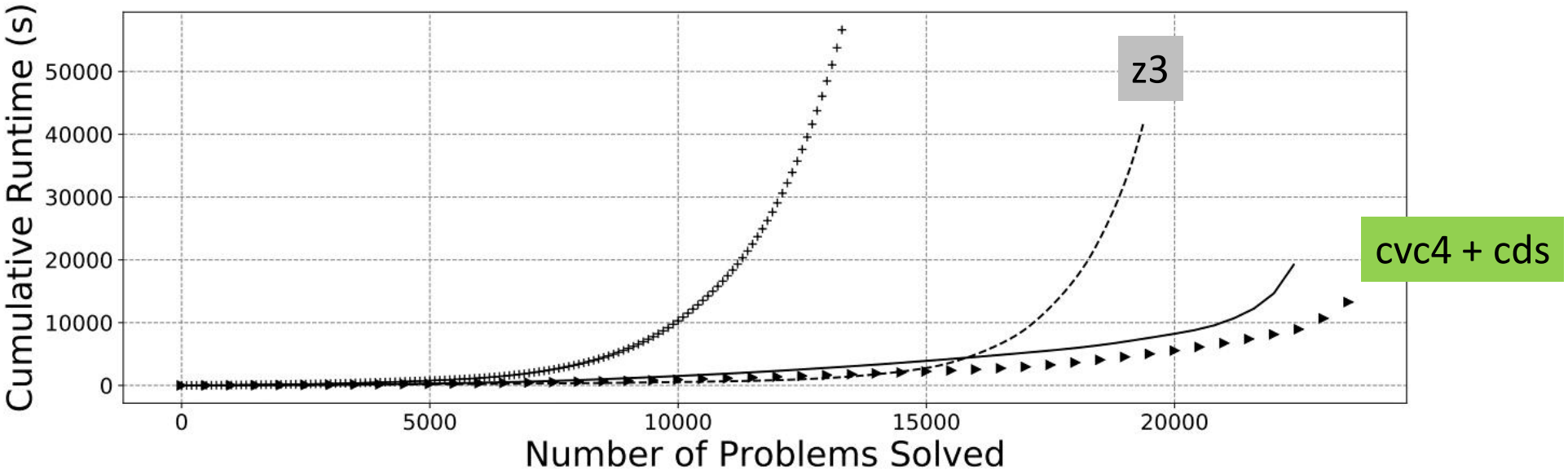
Experimental Results : PyEx Symbolic Execution

- Logged queries from PyEx symbolic execution engine (successor of PyExZ3)
  - Using z3str2, z3 and cvc4 as path constraint solver
- Total of 25,421 benchmarks over 3 runs
- Compared z3str2, z3, cvc4 w, w/o context-dependent simplification (cds)

Results : PyEx Symbolic Execution Benchmarks (25,421)

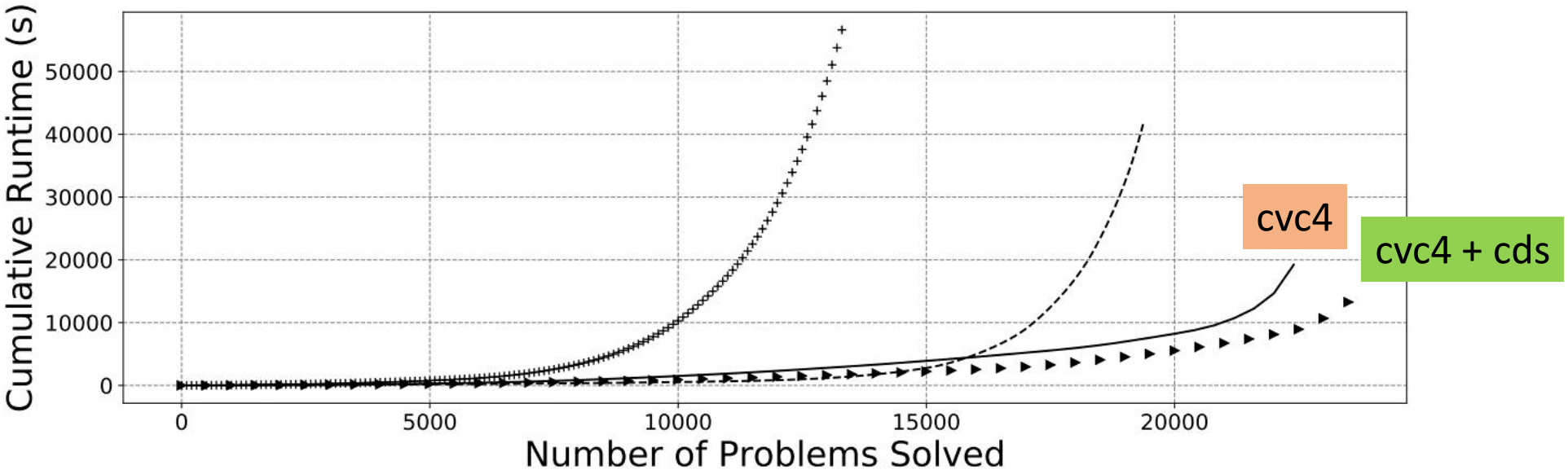


Results : PyEx Symbolic Execution Benchmarks (25,421)



- cvc4+context-dependent simplification solves 23,802 benchmarks in 5h8m
  - Nearest competitor z3 solves 19,368 benchmarks in 11h33m

Results : PyEx Symbolic Execution Benchmarks (25,421)



- By using context-dependent simplification:
  - cvc4+cds solves 536 benchmarks (+582 -46) w.r.t default cvc4
  - cvc4+cds expands 4.2x fewer extended terms per benchmark

Impact on PyEx Symbolic Execution

- Considered regression tests for 4 Python packages:
  - `httplib2`, `pip`, `pymongo`, `requests`
- Tested PyEx using different SMT backends:

Config	Time	Branch Coverage	Line Coverage
PyEx+z3str2	13h49m	3,500	8.34%
PyEx+z3	11h57m	**3,895**	8.41%
PyEx+cvc4	**4h55m**	3,612	**8.48%**

$\Rightarrow$ PyEx+cvc4 achieves comparable program coverage, much faster, wrt other solvers

Summary

- New technique context-dependent simplification implemented in CVC4's string solver
- Improves scalability on extended string constraints
- PyEx + CVC4 achieves comparable program coverage using 41% of the runtime as PyEx + nearest competitor

Future Work

- More aggressive simplification rules for strings
  - More powerful rules → better performance
- Quality of models for PyEx symbolic execution
  - Which models lead to higher code coverage?
- Apply context-dependent simplification to other theories:
  - Non-linear arithmetic
  - Lazy bit-blasting approaches to bit-vectors

  ⇒See **[Reynolds et al FroCoS 2017]** for details
- Simplification directly benefits Syntax-Guided Synthesis for strings in CVC4

- String solver in CVC4
  - Open source
  - Available at : http://cvc4.cs.stanford.edu/web/

- 25,421 new benchmarks from PyEx (*.smt2)
  - Available at : http://cvc4.cs.stanford.edu/papers/CAV2017-strings/

- …Thanks for listening!