Informatics 1

Functional Programming Lectures 15 and 16

IO and Monads

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Part I

The Mind-Body Problem

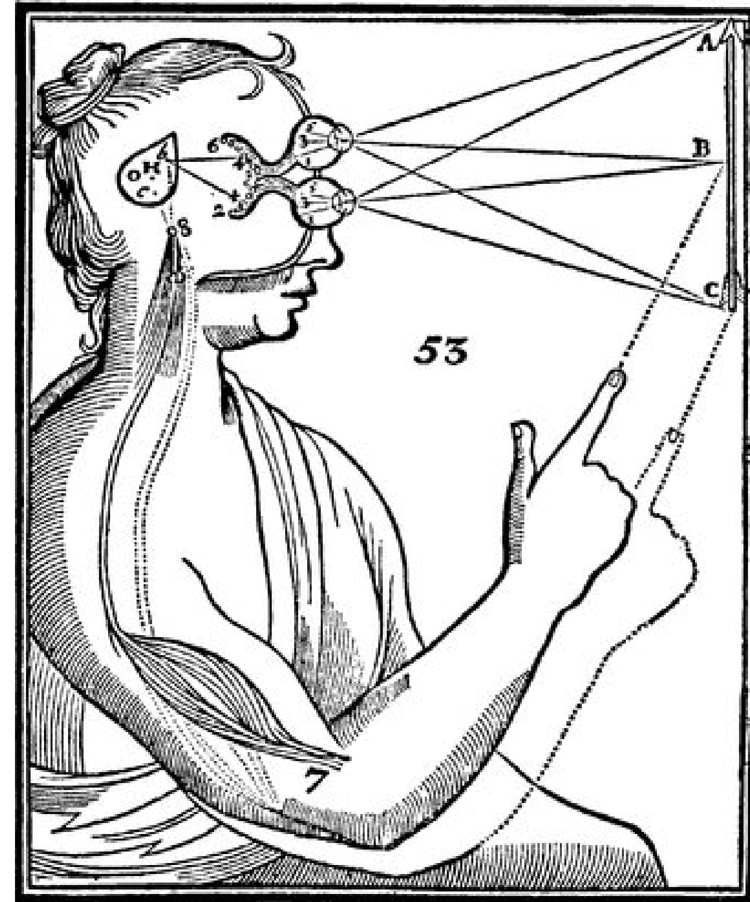
Everything in Haskell is "pure" in the sense of having no SIDE EFFECTS.
A side effect is something that happens "off to the side" during evaluation - input/output, change of state, etc.

A function in Java, C etc. might not be a real function - it might give different results when applied twice to the same value, e.g. because the result is dependent on input from the keyboard, or on the date. Then the order of evaluation matters.
If parts of the computation are done in parallel, then their relative timings can affect the result.

So side effects make things very complicated. But pure functions can be very inconvenient.
How do I get Haskell to DO SOMETHING - print, draw a picture, etc.?

The Mind-Body Problem

THE MECHANICAL PHILOSOPHY



Philosphers worried about the connection between THOUGHTS (mind) and ACTIONS (body). Where is the connection?
Descartes: the pineal gland (teardrop-shaped object in picture) is the connection between the soul (mind) and the brain/nerves (connected to sensory organs and muscles).
Haskell has something like a pineal gland, to make the connection between thinking (computation) and acting.

Part II

Commands

Print a character

```
putChar :: Char -> IO ()
```

For instance,

```
putChar '!'
```

denotes the command that, *if it is ever performed*, will print an exclamation mark.

Think of IO () as the type of COMMANDS.
() is the type of 0-tuples. The only value of type () is the 0-tuple, also written ().
putChar yields a command. It doesn't PERFORM the command, just PRODUCES it.
Production of commands is purely functional.

Combine two commands

```
(>>) :: IO () -> IO () -> IO ()
```

For instance,

```
putChar '?' >> putChar '!'
```

denotes the command that, *if it is ever performed*, prints a question mark followed by an exclamation mark.

>> (pronounced "then") combines two commands sequentially.
>> is associative but not commutative. Its identity is the command that does nothing.

Do nothing

```
done :: IO ()
```

The term `done` doesn't actually do nothing; it just specifies the command that, *if it is ever performed*, won't do anything. (Compare thinking about doing nothing to actually doing nothing: they are distinct enterprises.)

done is the identity for >>.

c >> done = c = done >> c

Print a string

```
putStr :: String -> IO ()
putStr []       =  done
putStr (x:xs)   =  putChar x >> putStr xs
```

So `putStr "?!"` is equivalent to

```
putChar '?' >> (putChar '!' >> done)
```

and both of these denote a command that, *if it is ever performed*, prints a question mark followed by an exclamation mark.

putStr produces a command that prints a string.

Higher-order functions

More compactly, we can define `putStr` as follows.

```
putStr  :: String -> IO ()
putStr  =  foldr (>>) done . map putChar
```

The operator >> has identity `done` and is associative.

```
m >> done         =  m
done >> m         =  m
(m >> n) >> o     =  m >> (n >> o)
```

Here's how we can define putStr more compactly.
You can use all of the features of Haskell to produce commands.

Main

By now you may be desperate to know *how is a command ever performed?* Here is the file `Confused.hs`:

```
module Confused where

main :: IO ()
main =  putStr "?!"
```

Running this program prints an indicator of perplexity:

```
[melchior]dts: runghc Confused.hs
?![melchior]dts:
```

Thus `main` is the link from Haskell's mind to Haskell's body — the analogue of Descartes's pineal gland.

This is enough, because we can make arbitrarily large combinations of things into a single command.

Print a string followed by a newline

```
putStrLn :: String -> IO ()
putStrLn xs  =  putStr xs >> putChar '\n'
```

Here is the file `ConfusedLn.hs`:

```
module ConfusedLn where

main :: IO ()
main =  putStrLn "?!"
```

This prints its result more neatly:

```
[melchior]dts: runghc ConfusedLn.hs
?!
[melchior]dts:
```

ghci evaluates expression and then uses show to display the resulting value.
For commands, show performs the command. So you don't need to use runghc.

Part III

Equational reasoning

Equational reasoning lost

In languages with side effects, this program prints "haha" as a side effect.

```
print "ha"; print "ha"
```

But this program only prints "ha" as a side effect.

```
let x = print "ha" in x; x
```

The value x of print "ha" is unimportant.
It's the side effect that matters.

This program again prints "haha" as a side effect.

```
let f () = print "ha" in f (); f ()
```

Each evaluation of f () produces the side effect.

Side effects make it harder to reason about programs.
Here is an example of what happens in a language (not Haskell) with side effects.

Equational reasoning regained

In Haskell, the term

```
(1+2) * (1+2)
```

and the term

```
let x = 1+2 in x * x
```

are equivalent (and both evaluate to 9).

In Haskell, the term

```
putStr "ha" >> putStr "ha"
```

and the term

```
let m = putStr "ha" in m >> m
```

are also entirely equivalent (and both print `"haha"`).

In Haskell, the simple equivalence rule works, even with commands that involve printing things.
You can always use a variable to factor out a common subexpression without changing the meaning.

Part IV

Commands with values

Read a character

Previously, we wrote `IO ()` for the type of commands that yield no value.

Here, `()` is the trivial type that contains just one value, which is also written `()`.

We write `IO Char` for the type of commands that yield a value of type `Char`.

Here is a command to read a character.

```
getChar :: IO Char
```

Performing the command `getChar` when the input contains `"abc"` yields the value `'a'` and remaining input `"bc"`.

Do nothing and return a value

More generally, we write `IO a` for commands that return a value of type `a`.

The command

```
return :: a -> IO a
```

is similar to `done`, in that it does nothing, but it also returns the given value.

Performing the command

```
return [] :: IO String
```

when the input contains `"bc"` yields the value `[]` and an unchanged input `"bc"`.

return is useful when combined with other commands - we'll see how in a minute.

Combining commands with values

We combine command with an operator written >>= and pronounced “bind”.

```
(>>=) :: IO a -> (a -> IO b) -> IO b
```

For example, performing the command

```
getChar >>= \x -> putChar (toUpper x)
```

when the input is `"abc"` produces the output `"A"`, and the remaining input is `"bc"`.

>>= is a generalisation of >> which handles the values produced when commands are performed.

The "bind" operator in detail

```
(>>=) :: IO a -> (a -> IO b) -> IO b
```

If

```
m :: IO a
```

is a command yielding a value of type `a`, and

```
k :: a -> IO b
```

is a function from a value of type `a` to a command yielding a value of type `b`, then

```
m >>= k :: IO b
```

is the command that, *if it is ever performed*, behaves as follows:

first perform command `m` yielding a value `x` of type `a`;
then perform command `k x` yielding a value `y` of type `b`;
then yield the final value `y`.

Performing this command performs m and then k.
The rest is about passing a value from m to k, and the fact that k is affected by the value passed.

Reading a line

Here is a program to read the input until a newline is encountered, and to return a list of the values read.

```
getLine :: IO String
getLine =  getChar >>= \x ->
           if x == '\n' then
             return []
           else
             getLine >>= \xs ->
             return (x:xs)
```

For example, given the input `"abc\ndef"` This returns the string `"abc"` and the remaining input is `"def"`.

1. Get a character, call it x.
2. if it's a new line, finished - return "".
3. otherwise, read the rest of the line (recursive call of getLine). This produces a string, call it xs.
4. return x:xs.

WIthout return, we couldn't write this!

Commands as a special case

The general operations on commands are:

```
return   :: a -> IO a
(>>=)    :: IO a -> (a -> IO b) -> IO b
```

The command `done` is a special case of `return`, and the operator `>>` is a special case of `>>=`.

```
done     :: IO ()
done     =  return ()

(>>)     :: IO () -> IO () -> IO ()
m >> n   =  m >>= \() -> n
```

An analogue of "let"

"let x = m in n" is another way of writing "n where x = m".

Although it may seem odd at first sight, this combinator is reassuringly similar to the familiar Haskell "let" expression. Here is a type rule for "let".

$$\frac{E \vdash \texttt{m :: a} \qquad E, \texttt{x :: a} \vdash \texttt{n :: b}}{E \vdash \textbf{let}\ \texttt{x = m}\ \textbf{in}\ \texttt{n :: b}}$$

m is an expression of type a

n is an expression of type b, with a variable x of type a.

then let x=m in n is an expression of type b

Typically, "bind" is combined with lambda expressions in a way that resembles "let" expressions. Here is the corresponding type rule.

$$\frac{E \vdash \texttt{m :: IO a} \qquad E, \texttt{x :: a} \vdash \texttt{n :: IO b}}{E \vdash \texttt{m >>= \textbackslash x -> n :: IO b}}$$

m returns a value of type a

n returns a value of type b, contains a variable of type a

m>>= \x->n does m, uses result to do n, returns value of type b

Echoing input to output

This program echoes its input to its output, putting everything in upper case, until an empty line is entered.

```
echo :: IO ()
echo =  getLine >>= \line ->
        if line == "" then
          return ()
        else
          putStrLn (map toUpper line) >>
          echo

main :: IO ()
main =  echo
```

1. Get a line of input, call it line.
2. If it's empty, we're done.
3. Otherwise, print the line in upper case.
4. Then do it again.

Testing it out

```
[melchior]dts: runghc Echo.hs
One line
ONE LINE
And, another line!
AND, ANOTHER LINE!
[melchior]dts:
```

Part V

“Do” notation

Reading a line in “do” notation

```
getLine :: IO String
getLine =  getChar >>= \x ->
           if x == '\n' then
             return []
           else
             getLine >>= \xs ->
             return (x:xs)
```

is equivalent to

```
getLine :: IO String
getLine =  do {
             x <- getChar;
             if x == '\n' then
               return []
             else do {
               xs <- getLine;
               return (x:xs)
             }
           }
```

This is a special notation that makes this easier.

do { x <- cmd; exp }   is the same as   cmd >>= \x -> exp

Echoing in “do” notation

```
echo :: IO ()
echo =  getLine >>= \line ->
        if line == "" then
          return ()
        else
          putStrLn (map toUpper line) >>
          echo
```

is equivalent to

```
echo :: IO ()
echo =  do {
          line <- getLine;
          if line == "" then
            return ()
          else do {
            putStrLn (map toUpper line);
            echo
          }
        }
```

do { cmd; exp } is the same as cmd >> exp which is short for cmd >>= \() -> exp

“Do” notation in general

Each line `x <- e; ...` becomes `e >>= \x -> ...`

Each line `e; ...` becomes `e >> ...`

For example,

```
do { x1 <- e1;
     x2 <- e2;
     e3;
     x4 <- e4;
     e5;
     e6 }
```

is equivalent to

```
e1 >>= \x1 ->
e2 >>= \x2 ->
e3 >>
e4 >>= \x4 ->
e5 >>
e6
```

Here is the general case. The result is the value returned by e6.

Part VI

Monads

Monoids

A *monoid* is a pair of an operator `(@@)` and a value `u`, where the operator has the value as identity and is associative.

```
u @@ x           =  x
x @@ u           =  x
(x @@ y) @@ z    =  x @@ (y @@ z)
```

Examples of monoids:

`(+)` and `0`

`(*)` and `1`

`(||)` and `False`

`(&&)` and `True`

`(++)` and `[]`

`(>>)` and `done`

Monads

We know that `(>>)` and `done` satisfy the laws of a *monoid*.

```
done >> m        =  m
m >> done        =  m
(m >> n) >> o    =  m >> (n >> o)
```

Similarly, `(>>=)` and `return` satisfy the laws of a *monad*.

```
return v >>= \x -> m         =  m[x:=v]
m >>= \x -> return x         =  m
(m >>= \x -> n) >>= \y-> o   =  m >>= \x -> (n >>= \y -> o)
```

A monad is a generalised version of a monoid.
The monad laws are the closest we can get to the monoid laws when >>= and return have the types they have.

Laws of Let

We know that `(>>)` and `done` satisfy the laws of a *monoid*.

```
done >> m          =  m
m >> done          =  m
(m >> n) >> o      =  m >> (n >> o)
```

Similarly, `(>>=)` and `return` satisfy the laws of a *monad*.

```
return v >>= \x -> m            =  m[x:=v]
m >>= \x -> return x            =  m
(m >>= \x -> n) >>= \y-> o      =  m >>= \x -> (n >>= \y -> o)
```

The three monad laws have analogues in “let” notation.

```
let x = v in m    =   m[x:=v]
let x = m in x    =   m
let y = (let x = m in n) in o
                  =   let x = m in (let y = n in o)
```

“Let” in languages with and without effects

```
let x = v in m    =   m[x:=v]
let x = m in x    =   m
let y = (let x = m in n) in o
                  =   let x = m in (let y = n in o)
```

These laws hold even in languages with side effects. For the first law to be true, `v` must be not an arbitrary term but a *value*, such as a constant or a variable (but not a function application). A value immediately evaluates to itself, hence it can have no side effects.

While in such languages one only has the above three laws for “let”, in Haskell one has a much stronger law, where one may replace a variable by any term, rather than by any value.

```
let x = n in m  =  m[x:=n]
```

Part VII

Roll your own monad—IO

The Monad type class

```
class Monad m where
  return :: a -> m a
  (>>=)  :: m a -> (a -> m b) -> m b
```

There is a type class for monads, and you can create instances yourselves.
Notice: in previous type classes, the variable (m here) referred to a TYPE.
But m here is a TYPE CONSTRUCTOR - a function that takes a type and yields a type.
We've seen the case where m is IO.

My own IO monad (1)

```
module MyIO(MyIO, myPutChar, myGetChar, convert) where

type Input = String
type Remainder = String
type Output = String

data MyIO a  =  MyIO (Input -> (a, Remainder, Output))

apply :: MyIO a -> Input -> (a, Remainder, Output)
apply (MyIO f) inp  =  f inp
```

Note that the type `MyIO` is abstract. The only operations on it are the monad operations, `myPutChar`, `myGetChar`, and `convert`. The operation `apply` is not exported from the module.

Here's how you can build your own version of IO.
Doing input/output, I take an input string and when I'm done I return a value, the unread part of the input string, and an output string.
MyIO is an abstract type - the constructor of MyIO is not exported.

My own IO monad (2)

```
myPutChar :: Char -> MyIO ()
myPutChar c  =  MyIO (\inp -> ((), inp, [c]))

myGetChar :: MyIO Char
myGetChar =  MyIO (\(ch:rem) -> (ch, rem, ""))
```

For example,

```
apply myGetChar "abc"  ==  ('a', "bc", "")
apply myGetChar "bc"   ==  ('b', "c", "")
apply (myPutChar 'A') "def"  ==  ((), "def", "A")
apply (myPutChar 'B') "def"  ==  ((), "def", "B")
```

myPutChar c takes input, returns no value, leaves input unchanged, and returns the string "c" as input.
myGetChar takes input, returns the first character, the remainder of the input is everything after that, and does no output.

My own IO monad (3)

```
instance Monad MyIO where
  return x   =  MyIO (\inp -> (x, inp, ""))
  m >>= k    =  MyIO (\inp ->
                  let (x, rem1, out1) = apply m inp in
                  let (y, rem2, out2) = apply (k x) rem1 in
                  (y, rem2, out1++out2))
```

I can put these things together to make a monad.

return x takes input, returns x, leaves input unchanged, does no output.

m >= k (1) applies m to input, getting value x, remaining input rem1, output out1
(2) applies k x to remaining input, getting value y, remaining input rem2, output out2
(3) result is y, remaining input is still rem2, output is concatenation of out1 and out2

For example

```
apply
  (myGetChar >>= \x -> myGetChar >>= \y -> return [x,y])
  "abc"
==  ("ab", "c", "")

apply
  (myPutChar 'A' >> myPutChar 'B')
  "def"
==  ((), "def", "AB")

apply
  (myGetChar >>= \x -> myPutChar (toUpper x))
  "abc"
== ((), "bc", "A")
```

My own IO monad (4)

```
convert :: MyIO () -> IO ()
convert m  =  interact (\inp ->
                 let (x, rem, out) = apply m inp in
                 out)
```

Here

```
interact :: (String -> String) -> IO ()
```

is part of the standard prelude. The entire input is converted to a string (lazily) and passed to the function, and the result from the function is printed as output (also lazily).

interact connects a function to actual input/output, making the connection to the keyboard and screen.

Using my own IO monad (1)

```
module MyEcho where

import Char
import MyIO

myPutStr :: String -> MyIO ()
myPutStr =  foldr (>>) (return ()) . map myPutChar

myPutStrLn :: String -> MyIO ()
myPutStrLn s  =  myPutStr s >> myPutChar '\n'
```

I can use this to build another version of the echo function.
First I versions of putStr and putStrLn for MyIO.

Using my own IO monad (2)

```
myGetLine :: MyIO String
myGetLine =  myGetChar >>= \x ->
             if x == '\n' then
               return []
             else
               myGetLine >>= \xs ->
               return (x:xs)

myEcho :: MyIO ()
myEcho =  myGetLine >>= \line ->
          if line == "" then
            return ()
          else
            myPutStrLn (map toUpper line) >>
            myEcho

main :: IO ()
main =  convert myEcho
```

Then I need a version of getLine for MyIO.
Finally, I write echo as before.
convert myEcho turns myEcho into IO ().

Trying it out

```
[melchior]dts: runghc MyEcho
This is a test.
THIS IS A TEST.
It is only a test.
IT IS ONLY A TEST.
Were this a real emergency, you'd be dead now.
WERE THIS A REAL EMERGENCY, YOU'D BE DEAD NOW.

[melchior]dts:
```

If you run it, the result is the same as before.
The only thing that wasn't pure Haskell functions and values was interact.

You can use “do” notation, too

```
myGetLine :: MyIO String
myGetLine =  do {
                x <- myGetChar;
                if x == '\n' then
                  return []
                else do {
                  xs <- myGetLine;
                  return (x:xs)
                }
              }

myEcho :: MyIO ()
myEcho =  do {
             line <- myGetLine;
             if line == "" then
               return ()
             else do {
               myPutStrLn (map toUpper line);
               myEcho
             }
           }
```

Because MyIO is a monad, you can use "do" notation.

Part VIII

The monad of lists

A lot of things turn out to have the structure of a monad.
We'll look first at lists, used to model non-deterministic computation.

The monad of lists

In the standard prelude:

```
class Monad m where
  return :: a -> m a
  (>>=)  :: m a -> (a -> m b) -> m b

instance Monad [] where

  return     :: a -> [a]
  return x   =  [ x ]

  (>>=)      :: [a] -> (a -> [b]) -> [b]
  m >>= k    =  [ y | x <- m, y <- k x ]
```

Think of a -> [b] modeling a non-deterministic computation
taking a value of type a
and returning a list of possible results of type b

return x: the only possible result is x

m >>= k: (1) do m, yielding a list of possible results of type a
(2) apply k to each of those possible results x
(3) result is a list containing all of (k x)'s results

Equivalently, we can define:

```
[] >>= k         =  []
(x:xs) >>= k     =  (k x) ++ (xs >>= k)
```

or

```
m >>= k   =  concat (map k m)
```

‘Do’ notation and the monad of lists

```
pairs :: Int -> [(Int, Int)]
pairs n  =  [ (i,j) | i <- [1..n], j <- [(i+1)..n] ]
```

is equivalent to

```
pairs' :: Int -> [(Int, Int)]
pairs' n  =  do {
                i <- [1..n];
                j <- [(i+1)..n];
                return (i,j)
              }
```

For example,

```
[melchior]dts: ghci Pairs
GHCi, version 6.10.4: http://www.haskell.org/ghc/  :? for help
Pairs> pairs 4
[(1,2),(1,3),(1,4),(2,3),(2,4),(3,4)]
Pairs> pairs' 4
[(1,2),(1,3),(1,4),(2,3),(2,4),(3,4)]
```

Notice that "do"-notation is now a lot like comprehension notation!

Monads with plus

In the standard prelude:

```
class Monad m => MonadPlus m where
  mzero :: m a
  mplus :: m a -> m a -> m a
```

Some monads have extra structure:
a plus operation and a zero value
that is the identity for plus.
MonadPlus is a monad with these extra things.

```
instance MonadPlus [] where

  mzero  :: [a]
  mzero  =  []

  mplus  :: [a] -> [a] -> [a]
  mplus  =  (++)
```

```
guard :: MonadPlus m => Bool -> m ()
guard False  =  mzero
guard True   =  return ()
```

Non-deterministic interpretation:
mzero = [] = no possible result
return () = [()] = one possible result

```
msum :: MonadPlus m => [m a] -> m a
msum =  foldr mplus mzero
```

msum is a generalisation of concat

Using guards

```
pairs'' :: Int -> [(Int, Int)]
pairs'' n  =  [ (i,j) | i <- [1..n], j <- [1..n], i < j ]
```

is equivalent to

```
pairs''' :: Int -> [(Int, Int)]
pairs''' n  =  do {
                  i <- [1..n];
                  j <- [1..n];
                  guard (i < j);
                  return (i,j)
                }
```

The effect of guard (i < j) is to drop the computation or continue.
Think of it as z <- guard (i < j):
if False, there is no z so we can't continue
if True, z=() and we continue (but don't use z)

For example,

```
[melchior]dts: ghci Pairs
GHCi, version 6.10.4: http://www.haskell.org/ghc/  :? for help
Pairs> pairs'' 4
[(1,2),(1,3),(1,4),(2,3),(2,4),(3,4)]
Pairs> pairs''' 4
[(1,2),(1,3),(1,4),(2,3),(2,4),(3,4)]
```

Now we have all of comprehension notation arising via "do"-notation in a monad with plus.

Part IX

Parsers

We will use monads to build a parser for expression trees.

Parser type

First attempt:

A parser for values of type a takes a string and produces a value of type a

```
type Parser a = String -> a
```

Second attempt:

But at each stage, you have a part that you've parsed and a remainder that is left to parse.

```
type Parser a = String -> (a, String)
```

Third attempt:

But sometimes there will be more than one parse, or none. So you need a list of possibilities.

```
type Parser a = String -> [(a, String)]
```

A parser for things
is a function from strings
to lists of pairs
Of things and strings

—Graham Hutton

Module Parser

```
module Parser(Parser,apply,parse,char,spot,token,
  star,plus,parseInt) where

import Char
import Monad

-- The type of parsers
data Parser a = Parser (String -> [(a, String)])

-- Apply a parser
apply :: Parser a -> String -> [(a, String)]
apply (Parser f) s  =  f s

-- Return parsed value, assuming at least one successful parse
parse :: Parser a -> String -> a
parse m s  =  head [ x | (x,t) <- apply m s, t == "" ]
```

Parser is an abstract data type - the constructor is not exported.
There is an implicit invariant: the string produced is always a final substring of the string consumed.
parse m s looks for a result (x,t) from apply m s which completely consumes s. We take the first one we find.
(If there is more than one, ambiguous - we could return an error.)

Parser is a Monad

```
-- Parsers form a monad

--    class Monad m where
--      return :: a -> m a
--      (>>=) :: m a -> (a -> m b) -> m b

instance Monad Parser where
  return x  =  Parser (\s -> [(x,s)])
  m >>= k   =  Parser (\s ->
                 [ (y, u) |
                   (x, t) <- apply m s,
                   (y, u) <- apply (k x) t ])
```

Parsers form a monad

return x: just return x, without consuming any input

m >>= k:
(1) apply m to s, giving possibilities (x,t) where t is the unparsed remainder
(2) apply (k x) to t, giving possibilities (y,u) where u is the final unparsed remainder
(3) the result is all of the possible (y,u) resulting from all of the possible (x,t)

Parser is a Monad with Plus

```
-- Some monads have additional structure

--    class MonadPlus m where
--      mzero :: m a
--      mplus :: m a -> m a -> m a

instance MonadPlus Parser where
  mzero       =  Parser (\s -> [])
  mplus m n   =  Parser (\s -> apply m s ++ apply n s)
```

Parsers also form a monad with plus.
The monad structure gives SEQUENCING.
The plus structure gives ALTERNATIVES.

Parsing characters

```
-- Parse a single character
char :: Parser Char
char =  Parser f
  where
  f []      =  []
  f (c:s)   =  [(c,s)]
```

Here is a parser that parses the first character in the input.
If input is empty, failure: empty list of possibilities.
If input is non-empty, first character is result, tail is remaining string.

```
-- Parse a character satisfying a predicate (e.g., isDigit)
spot :: (Char -> Bool) -> Parser Char
spot p  =  Parser f
  where
  f []                   =  []
  f (c:s) | p c          =  [(c, s)]
          | otherwise    =  []
```

This is similar, but requires the character to satisfy a given predicate.
So spot isDigit will only parse digits.

```
-- Parse a given character
token :: Char -> Parser Char
token c  =  spot (== c)
```

token c will parse the character c only.
It will fail if the next character is something else.

Parsing characters

```
-- Parse a single character
char :: Parser Char
char =  Parser f
  where
  f []     =  []
  f (c:s)  =  [(c,s)]

-- Parse a character satisfying a predicate (e.g., isDigit)
spot :: (Char -> Bool) -> Parser Char
spot p  =  do { c <- char; guard (p c); return c }
```

Here's the same thing, using do-notation and guard; remember, guard is defined for any MonadPlus.

```
-- Parse a given character
token :: Char -> Parser Char
token c  =  spot (== c)
```

Examples:
apply (spot isDigit) "123ab" = [('1', "23ab")]
apply (spot isDigit) "ab" = []
apply (token 'a') "ab" = [('a', "b")]
apply (do {x <- spot isDigit; token '+'; y <- spot isDigit; return (digitToInt x + digitToInt y)}) "1+2*4" = [(3, "*4"])

Parsing a string

```
match :: String -> Parser String
match []       =  return []
match (x:xs)   =  do
                    y <- token x;
                    ys <- match xs;
                    return (y:ys)
```

To parse a whole string:

if it is empty, you're done - return ""

if it is x:xs

(1) y is the result of token x (y = x, but token x will consume a character from the input and fail if it isn't x)

(2) then parse xs, yielding ys (again, ys = xs)

(3) then return y:ys

Parsing a sequence

```
-- match zero or more occurrences
star :: Parser a -> Parser [a]
star p  =  plus p `mplus` return []

-- match one or more occurrences
plus :: Parser a -> Parser [a]
plus p  =  do { x <- p;
                xs <- star p;
                return (x:xs) }
```

star p is like p* from regular expressions: it parses zero or more occurrences of p.
plus p parses one or more occurrences of p.
Note the mutual recursion.

Example:
apply (star (spot isDigit)) "123ab" = [("123", "ab"), ("12", "3ab"), ("1", "23ab"), ("", "123ab")]

Parsing an integer

```
-- match a natural number
parseNat :: Parser Int
parseNat =  do { s <- plus (spot isDigit);
                 return (read s) }
```

parse one or more digits
then convert it to an Int

```
-- match a negative number
parseNeg :: Parser Int
parseNeg =  do { token '-';
                 n <- parseNat
                 return (-n) }
```

parse a minus sign followed by a natural number

```
-- match an integer
parseInt :: Parser Int
parseInt =  parseNat `mplus` parseNeg
```

parse an integer: parse either a positive natural number or a negative one

Module Exp

```
module Exp where

import Monad
import Parser

data Exp = Lit Int
         | Exp :+: Exp
         | Exp :*: Exp
         deriving (Eq,Show)

evalExp :: Exp -> Int
evalExp (Lit n)     =  n
evalExp (e :+: f)   =  evalExp e + evalExp f
evalExp (e :*: f)   =  evalExp e * evalExp f
```

Here's Exp (expression trees) from before, with an evaluation function.

Parsing an expression

```
parseExp :: Parser Exp
parseExp  =  parseLit `mplus` parseAdd `mplus` parseMul
  where
  parseLit = do { n <- parseInt;
                  return (Lit n) }
  parseAdd = do { token '(';
                  d <- parseExp;
                  token '+';
                  e <- parseExp;
                  token ')';
                  return (d :+: e) }
  parseMul = do { token '(';
                  d <- parseExp;
                  token '*';
                  e <- parseExp;
                  token ')';
                  return (d :*: e) }
```

An Exp is either a literal, or an addition, or a multiplication

Literal: parse an integer n, return Lit n

Addition:
(1) parse (
(2) parse an Exp d
(3) parse +
(4) parse an Exp e
(5) parse)
(6) return d :+: e

Multiplication: similarly

Notice: after you've read (Exp you don't know if the next thing will be + or *. mplus handles that.
Also, expressions can be nested. Recursion handles that.

Testing the parser

```
[melchior]dts: ghci Exp.hs
GHCi, version 6.10.4: http://www.haskell.org/ghc/  :? for help
[1 of 2] Compiling Parser           ( Parser.hs, interpreted )
[2 of 2] Compiling Exp              ( Exp.hs, interpreted )
Ok, modules loaded: Parser, Exp.
*Exp> parse parseExp "(1+(2*3))"
Lit 1 :+: (Lit 2 :*: Lit 3)
*Exp> evalExp (parse parseExp "(1+(2*3))")
7
*Exp> parse parseExp "((1+2)*3)"
(Lit 1 :+: Lit 2) :*: Lit 3
*Exp> evalExp (parse parseExp "((1+2)*3)")
9
*Exp>
```

And it works! We can parse strings to give Exp values, which can be evaluated.